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Chapter 4

INDUCTION MACHINE WINDINGS AND THEIR M.M.Fs

4.1. INTRODUCTION

As shown in Chapter 2, the slots of the stator and rotor cores of induction machines are filled with electric conductors, insulated (in the stator) from cores, and connected in a certain way. This ensemble constitutes the windings. The primary (or the stator) slots contain a polyphase (triple phase or double phase) a.c. winding. The rotor may have either a 3(2) phase winding or a squirrel cage. Here we will discuss the polyphase windings.

Designing a.c. windings means, in fact, assigning coils in the slots to various phases, establishing the direction of currents in coil sides and coil connections per phase and between phases, and finally calculating the number of turns for various coils and the conductor sizing.

We start with single pole number three-phase windings as they are most commonly used in induction motors. Then pole changing windings are treated in some detail. Such windings are used in wind generators or in doubly fed variable speed configurations. Two phase windings are given special attention. Finally, squirrel cage winding m.m.fs are analyzed.

Keeping in mind that a.c. windings are a complex subject having books dedicated to it [1,2] we will treat here first its basics. Then we introduce new topics such as “pole amplitude modulation,” “polyphase symmetrization” [4], “intersperse windings” [5], “simulated annealing” [7], and “the three-equation principle” [6] for pole changing. These are new ways to produce a.c. windings for special applications (for pole changing or m.m.f. chosen harmonics elimination). Finally, fractional multilayer three-phase windings with reduced harmonics content are treated in some detail [8,9]. The present chapter is structured to cover both the theory and case studies of a.c. winding design, classifications, and magnetomotive force (mmf) harmonic analysis.

4.2. THE IDEAL TRAVELING M.M.F. OF A.C. WINDINGS

The primary (a.c. fed) winding is formed by interconnecting various conductors in slots around the circumferential periphery of the machine. As shown in Chapter 2, we may have a polyphase winding on the so-called wound rotor. Otherwise, the rotor may have a squirrel cage in its slots. The objective with polyphase a.c. windings is to produce a pure traveling m.m.f., through proper feeding of various phases with sinusoidal symmetrical currents. And all this in order to produce constant (rippleless) torque under steady state:

$$F_{st}(x, t) = F_{slm} \cos\left(\frac{\pi}{\tau}x - \omega_1 t - \theta_0\right) \quad (4.1)$$

where

x - coordinate along stator bore periphery

τ - spatial half-period of m.m.f. ideal wave

ω_1 - angular frequency of phase currents

θ_0 - angular position at $t = 0$

We may decompose (4.1) into two terms

$$F_{sl}(x, t) = F_{slm} \left[\cos\left(\frac{\pi}{\tau}x - \theta_0\right) \cos\omega_1 t + \sin\left(\frac{\pi}{\tau}x - \theta_0\right) \sin\omega_1 t \right] \quad (4.2)$$

Equation (4.2) has a special physical meaning. In essence, there are now two mmfs at standstill (fixed) with sinusoidal spatial distribution and sinusoidal currents. The space angle lag and the time angle between the two mmfs is $\pi/2$. This suggests that a pure traveling mmf may be produced with two symmetrical windings $\pi/2$ shifted in time (Figure 4.1.a). This is how the two phase induction machine evolved.

Similarly, we may decompose (4.1) into 3 terms

$$F_{sl}(x, t) = \frac{2}{3} F_{slm} \left[\cos\left(\frac{\pi}{\tau}x - \theta_0\right) \cos\omega_1 t + \cos\left(\frac{\pi}{\tau}x - \theta_0 - \frac{2\pi}{3}\right) \cos\left(\omega_1 t - \frac{2\pi}{3}\right) + \cos\left(\frac{\pi}{\tau}x - \theta_0 + \frac{2\pi}{3}\right) \cos\left(\omega_1 t + \frac{2\pi}{3}\right) \right] \quad (4.3)$$

Consequently, three mmfs (single-phase windings) at standstill (fixed) with sinusoidal spatial (x) distribution and departed in space by $2\pi/m$ radians, with sinusoidal symmetrical currents—equal amplitude, $2\pi/3$ radians time lag angle—are also able to produce also a traveling mmf (Figure 4.1.b).

In general, m phases with a phase lag (in time and space) of $2\pi/3$ can produce a traveling wave. Six phases ($m = 6$) would be a rather practical case besides $m = 3$ phases. The number of mmf electrical periods per one revolution is called the number of pole pairs p_1

$$p_1 = \frac{\pi D}{2\tau}; \quad 2p_1 = 2, 4, 6, 8, \dots \quad (4.4)$$

where D is the stator bore diameter.

It should be noted that, for $p_1 > 1$, according to (4.4), the electrical angle α_e is p_1 times larger than the mechanical angle α_g

$$\alpha_e = p_1 \alpha_g \quad (4.5)$$

A sinusoidal distribution of mmfs (ampere-turns) would be feasible only with the slotless machine and windings placed in the airgap. Such a solution is hardly practical for induction machines because the magnetization of a large

total airgap would mean very large magnetization mmf and, consequently, low power factor and efficiency. It would also mean problems with severe mechanical stress acting directly on the electrical conductors of the windings.

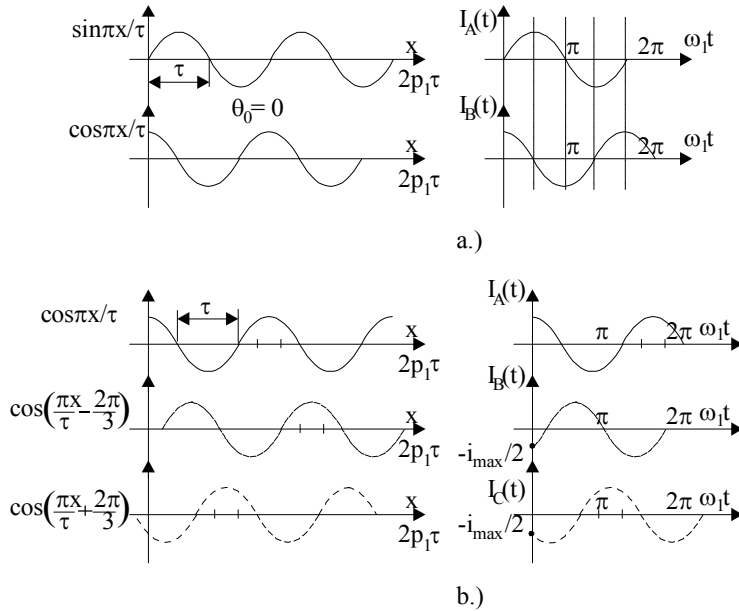


Figure 4.1 Ideal multiphase mmfs
a.) two-phase machine b.) three-phase machine

In practical induction machines, the coils of the windings are always placed in slots of various shapes (Chapter 2).

The total number of slots per stator N_s should be divisible by the number of phases m so that

$$N_s / m = \text{integer} \quad (4.6)$$

A parameter of great importance is the number of slots per pole per phase q :

$$q = \frac{N_s}{2p_1 m} \quad (4.7)$$

The number q may be an integer ($q = 1, 2, \dots, 12$) or a fraction.

In most induction machines, q is an integer to provide complete (pole to pole) symmetry for the winding.

The windings are made of coils. Lap and wave coils are used for induction machines (Figure 4.2).

The coils may be placed in slots in one layer (Figure 4.2a) or in two layers (Figure 4.3.b).

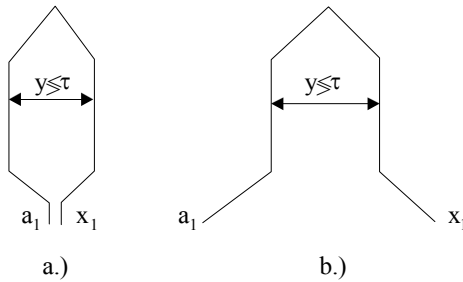


Figure 4.2 Lap a.) and wave b.) single-turn (bar) coils

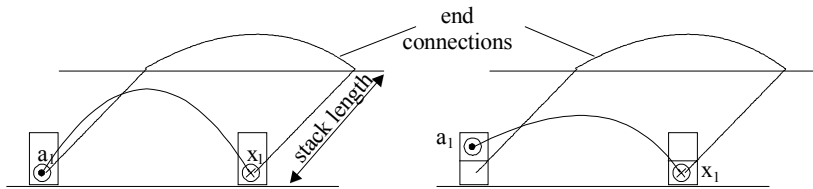


Figure 4.3 Single-layer a.) and double-layer b.) coils (windings)

Single layer windings imply full pitch ($y = \tau$) coils to produce an mmf fundamental with pole pitch τ .

Double layer windings also allow chorded (or fractional pitch) coils ($y < \tau$) such that the end connections of coils are shortened and thus copper loss is reduced. Moreover, as shown later in this chapter, the harmonics content of mmf may be reduced by chorded coils. Unfortunately, so is the fundamental.

4.3. A PRIMITIVE SINGLE-LAYER WINDING

Let us design a four pole ($2p_1 = 4$) three-phase single-layer winding with $q = 1$ slots/pole/phase. $N_s = 2p_1 q m = 2 \cdot 2 \cdot 1 \cdot 3 = 12$ slots in all.

From the previous paragraph, we infer that for each phase we have to produce an mmf with $2p_1 = 4$ poles (semiperiods). To do so, for a single layer winding, the coil pitch $y = \tau = N_s / 2p_1 = 12 / 4 = 3$ slot pitches.

For 12 slots there are 6 coils in all. That is, two coils per phase to produce 4 poles. It is now obvious that the 4 phase A slots are $y = \tau = 3$ slot pitches apart. We may start in slot 1 and continue with slots 4, 7, and 10 for phase A ([Figure 4.4a](#)).

Phases B and C are placed in slots by moving $2/3$ of a pole (2 slots pitches in our case) to the right. All coils/phases may be connected in series to form one current path ($a = 1$) or they may be connected in parallel to form two current paths in parallel ($a = 2$). The number of current paths a is obtained in general by connecting part of coils in series and then the current paths in parallel such that all the current paths are symmetric. Current paths in parallel serve to reduce wire gauge (for given output phase current) and, as shown later, to reduce

uncompensated magnetic pull between rotor and stator in presence of rotor eccentricity.

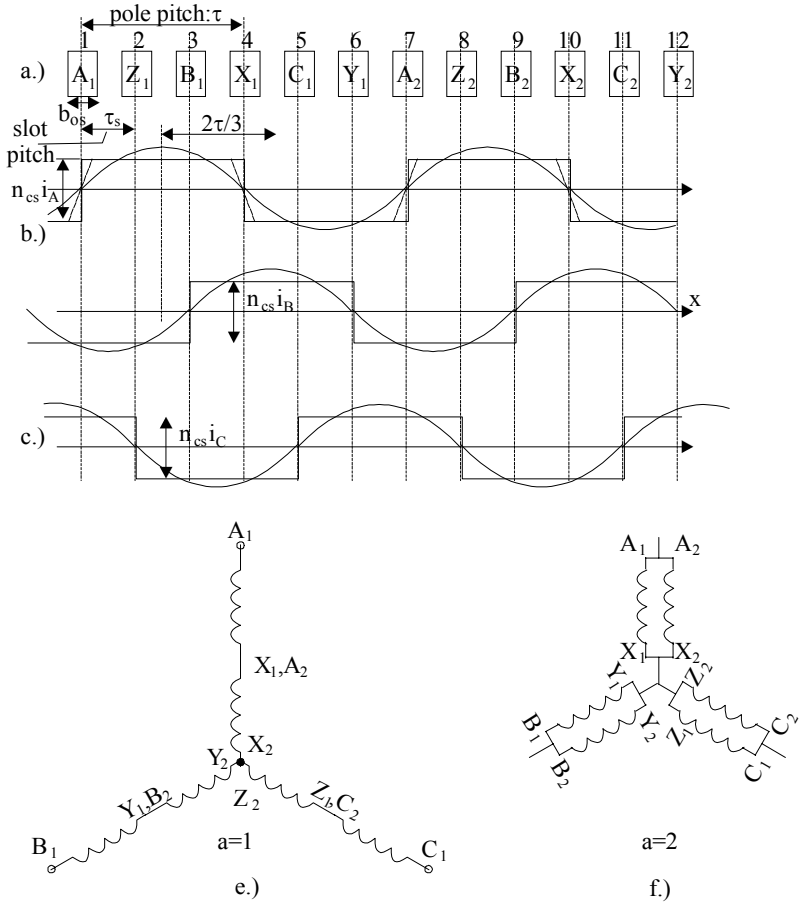


Figure 4.4 Single-layer three-phase winding for $2p_1 = 4$ poles and $q = 1$ slots/pole/phase: a.) slot/phase allocation; b.), c.), d.) ideal mmf distribution for the three phases when their currents are maximum; e.) star series connection of coils/phase; f.) parallel connection of coils/phase

If the slot is considered infinitely thin (or the slot opening $b_{os} \approx 0$), the mmf (ampereturns) jumps, as expected, by $n_{cs} i_{A,B,C}$ along the middle of each slot.

For the time being, let us consider $b_{os} = 0$ (a virtual closed slot).

The rectangular mmf distribution may be decomposed into harmonics for each phase. For phase A we simply obtain

$$F_{A1}(x, t) = \frac{2}{\pi} \cdot \frac{n_{cs} I \sqrt{2} \cos \omega_1 t}{v} \cos \frac{v \pi x}{\tau} \quad (4.8)$$

For the fundamental, $v = 1$, we obtain the maximum amplitude. The higher the order of the harmonic, the lower its amplitude in (4.8).

While in principle such a primitive machine works, the harmonics content is too rich.

It is only intuitive that if the number of steps in the rectangular ideal distribution would be increased, the harmonics content would be reduced. This goal could be met by increasing q or (and) via chording the coils in a two-layer winding. Let us then present such a case.

4.4. A PRIMITIVE TWO-LAYER CHORDED WINDING

Let us still consider $2p_1 = 4$ poles, $m = 3$ phases, but increase q from 1 to 2. Thus the total number of slots $N_s = 2p_1 q m = 2 \cdot 2 \cdot 2 \cdot 3 = 24$.

The pole pitch τ measured in slot pitches is $\tau = N_s / 2p_1 = 24 / 4 = 6$. Let us reduce the coil throw (span) y such that $y = 5\tau / 6$.

We still have to produce 4 poles. Let us proceed as in the previous paragraph but only for one layer, disregarding the coil throw.

In a two, layer winding, the total number of coils is equal to the number of slots. So in our case there are $N_s / m = 24 / 3$ coils per phase. Also, there are 8 slots occupied by one phase in each layer, four with inward and four with outward current direction. With each layer each phase has to produce four poles in our case. So slots 1, 2; 7', 8'; 13, 14; 19', 20' in layer one belong to phase A. The superscript prime refers to outward current direction in the coils. The distance between neighbouring slot groups of each phase in one layer is always equal to the pole pitch to preserve the mmf distribution half-period (Figure 4.5).

Notice that in Figure 4.5, for each phase, the second layer is displaced to the left by $\tau - y = 6 - 5 = 1$ slot pitch with respect to the first layer. Also, after two poles, the situation repeats itself. This is typical for a fully symmetrical winding.

Each coil has one side in one layer, say, in slot 1, and the second one in slot $y + 1 = 5 + 1 = 6$. In this case all coils are identical and thus the end connections occupy less axial room and are shorter due to chording. Such a winding is typical with random wound coils made of round magnetic wire.

For this case we explore the mmf ideal resultant distribution for the situation when the current in phase A is maximum ($i_A = i_{\max}$). For symmetrical currents, $i_B = i_C = -i_{\max} / 2$ (Figure 4.1b).

Each coil has n_c conductors and, again with zero slot opening, the mmf jumps at every slot location by the total number of ampereturns. Notice that half the slots have coils of same phase while the other half accommodate coils of different phases.

The mmf of phase A, for maximum current value (Figure 4.5b) has two steps per polarity as $q = 2$. It had only one step for $q = 1$ (Figure 4.4). Also, the resultant mmf has three unequal steps per polarity ($q + \tau - y = 2 + 6 - 5 = 3$). It is indeed closer to a sinusoidal distribution. Increasing q and using chorded coils reduces the harmonics content of the mmf.

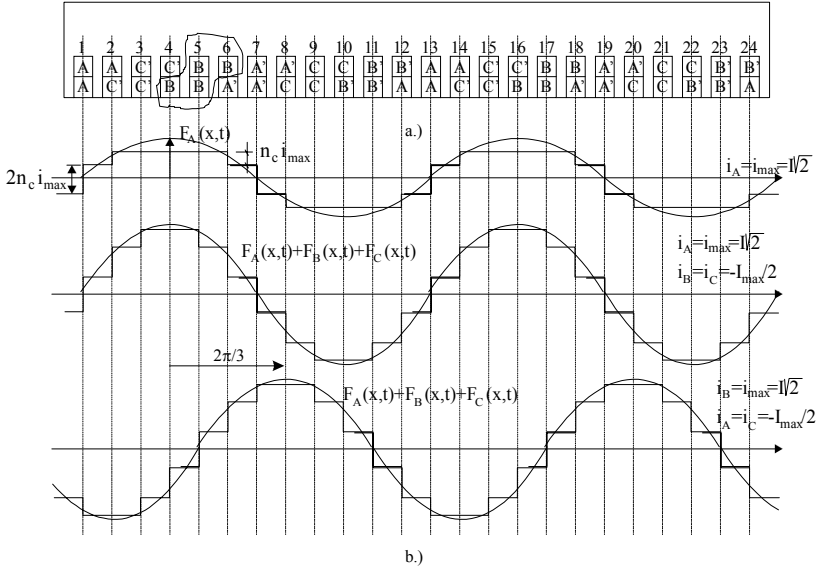


Figure 4.5 Two-layer winding for $N_s = 24$ slots, $2 p_1 = 4$ poles, $y/\tau = 5/6$
a.) slot/phase allocation, b.) mmfs distribution

Also shown in [Figure 4.5](#) is the movement by $2\pi/3$ (or $2\pi/3$ electrical radians) of the mmf maximum when the time advances with $2\pi/3$ electrical (time) radians or $T/3$ (T is the time period of sinusoidal currents).

4.5. THE MMF HARMONICS FOR INTEGER q

Using the geometrical representation in [Figure 4.5](#), it becomes fairly easy to decompose the resultant mmf in harmonics noticing the step-form of the distributions.

Proceeding with phase A we obtain (by some extrapolation for integer q),

$$F_{A1}(x, t) = \frac{2}{\pi} n_c q I \sqrt{2} K_{q1} K_{y1} \cos \frac{\pi}{\tau} x \cos \omega_1 t \quad (4.9)$$

$$\text{with} \quad K_{q1} = \sin \pi / 6 / (q \sin \pi / 6q) \leq 1; K_{y1} = \sin \frac{\pi}{2} y / \tau \leq 1 \quad (4.10)$$

K_{q1} is known as the zone (or spread) factor and K_{y1} the chording factor. For $q = 1$, $K_{q1} = 1$ and for full pitch coils, $y/\tau = 1$, $K_{y1} = 1$, as expected.

To keep the winding fully symmetric $y/\tau \geq 2/3$. This way all poles have a similar slot/phase allocation.

Assuming now that all coils per phase are in series, the number of turns per phase W_1 is

$$W_l = 2p_l q n_c \quad (4.11)$$

With (4.11), Equation (4.9) becomes

$$F_{A1}(x, t) = \frac{2}{\pi p_l} W_l I \sqrt{2} K_{ql} K_{yl} \cos \frac{\pi}{\tau} x \cos \omega_l t \quad (4.12)$$

For three phases we obtain

$$F_l(x, t) = F_{lm} \cos \left(\frac{\pi}{\tau} x - \omega_l t \right) \quad (4.13)$$

with
$$F_{lm} = \frac{3W_l I \sqrt{2} K_{ql} K_{yl}}{\pi p_l} \text{ (ampereturns per pole)} \quad (4.14)$$

The derivative of pole mmf with respect to position x is called linear current density (or current sheet) A (in Amps/meter)

$$A_l(x, t) = \frac{\partial F_l(x, t)}{\partial x} = A_{lm} \sin \left(-\frac{\pi}{\tau} x + \omega_l t \right) \quad (4.15)$$

$$A_{lm} = \frac{3\sqrt{2} W_l I \sqrt{2} K_{ql} K_{yl}}{p_l \tau} = \frac{\pi}{\tau} F_{lm} \quad (4.16)$$

A_{lm} is the maximum value of the current sheet and is also identified as current loading. The current loading is a design parameter (constant) $A_{lm} \approx 5,000 \text{ A/m}$ to $50,000 \text{ A/m}$, in general, for induction machines in the power range of kilowatts to megawatts. It is limited by the temperature rise and increases with machine torque (size).

The harmonics content of the mmf is treated in a similar manner to obtain

$$F(x, t) = \frac{3W_l I \sqrt{2} K_{qv} K_{yv}}{\pi p_l v} \cdot \left[K_{Bl} \cos \left(\frac{v\pi}{\tau} x - \omega_l t - (v-1) \frac{2\pi}{3} \right) - K_{BII} \cos \left(\frac{v\pi}{\tau} x + \omega_l t - (v+1) \frac{2\pi}{3} \right) \right] \quad (4.17)$$

with

$$K_{qv} = \frac{\sin v\pi/6}{q \sin v\pi/6q}; K_{yv} = \sin \left(\frac{v\pi y}{2\tau} \right) \quad (4.18)$$

$$K_{Bl} = \frac{\sin(v-1)\pi}{3 \sin(v-1)\pi/3}; K_{BII} = \frac{\sin(v+1)\pi}{3 \sin(v+1)\pi/3} \quad (4.19)$$

Due to mmf full symmetry (with $q = \text{integer}$), only odd harmonics occur. For three-phase star connection, $3K$ harmonics may not occur as the current sum is zero and their phase shift angle is $3K \cdot 2\pi/3 = 2\pi K$.

We are left with harmonics $v = 3K \pm 1$; that is $v = 5, 7, 11, 13, 17, \dots$.

We should notice in (4.19) that for $v_d = 3K + 1$, $K_{BI} = 1$ and $K_{BII} = 0$. The first term in (4.17) represents however a direct (forward) traveling wave as for a constant argument under cosinus, we do obtain

$$\left(\frac{dx}{dt} \right) = \frac{\omega_1 \tau}{\pi v} = \frac{2\tau f_1}{v}; \omega_1 = 2\pi f_1 \quad (4.20)$$

On the contrary, for $v = 3K-1$, $K_{BI} = 0$, and $K_{BII} = 1$. The second term in (4.17) represents a backward traveling wave. For a constant argument under cosinus, after a time derivative, we have

$$\left(\frac{dx}{dt} \right)_{v=3K-1} = \frac{-\omega_1 \tau}{\pi v} = \frac{-2\tau f_1}{v} \quad (4.21)$$

We should also notice that the traveling speed of mmf space harmonics, due to the placement of conductors in slots, is v times smaller than that of the fundamental ($v = 1$).

The space harmonics of the mmf just investigated are due both to the placement of conductors in slots and to the placement of various phases as phase belts under each pole. In our case the phase belts spread is $\pi/3$ (or one third of a pole). There are also two layer windings with $2\pi/3$ phase belts but the $\pi/3$ (60°) phase belt windings are more practical.

So far the slot opening influences on the mmf stepwise distribution have not been considered. It will be discussed later in this chapter.

Notice that the product of zone (spread or distribution) factor K_{qv} and the chording factor K_{yv} is called the stator winding factor K_{wv} .

$$K_{wv} = K_{qv} K_{yv} \quad (4.22)$$

As in most cases, only the mmf fundamental ($v = 1$) is useful, reducing most harmonics and cancelling some is a good design attribute. Chording the coils to cancel K_{yv} leads to

$$\sin\left(\frac{v\pi y}{2\tau}\right) = 0; \frac{v\pi y}{2\tau} = n\pi; \frac{y}{\tau} > \frac{2}{3} \quad (4.23)$$

As the mmf harmonic amplitude (4.17) is inversely proportional to the harmonic order, it is almost standard to reduce (cancel) the fifth harmonic ($v = 5$) by making $n = 2$ in (4.23).

$$\frac{y}{\tau} = \frac{4}{5} \quad (4.23')$$

In reality, this ratio may not be realized with an integer q ($q = 2$) and thus $y/\tau = 5/6$ or $7/9$ is the practical solution which keeps the 5th mmf harmonic low.

Chording the coils also reduces K_{y1} . For $y/\tau = 5/6$, $\sin \frac{\pi}{2} \frac{5}{6} = 0.966 < 1.0$ but a

4% reduction in the mmf fundamental is worth the advantages of reducing the coil end connection length (lower copper losses) and a drastical reduction of 5th mmf harmonic.

Mmf harmonics, as will be shown later in the book, produce parasitic torques, radial forces, additional core and winding losses, noise, and vibration.

Example 4.1.

Let us consider an induction machine with the following data: stator core diameter $D = 0.1$ m, number of stator slots $N_s = 24$, number of poles $2p_1 = 4$, $y/\tau = 5/6$, two-layer winding; slot area $A_{\text{slot}} = 100 \text{ mm}^2$, total slot fill factor $K_{\text{fill}} = 0.5$, current density $j_{\text{Co}} = 5 \text{ A/mm}^2$, number of turns per coil $n_c = 25$. Let us calculate

- The rated current (RMS value), wire gauge
- The pole pitch τ
- K_{q1} and K_{y1} , K_{w1}
- The amplitude of the mmf F_{1m} and of the current sheet A_{1m}
- K_{q7} , K_{y7} and F_{7m} ($v = 7$)

Solution

Part of the slot is filled with insulation (conductor insulation, slot wall insulation, layer insulation) because there is some room between round wires. The total filling factor of a slot takes care of all these aspects. The mmf per slot is

$$2n_c I = A_{\text{slot}} \cdot K_{\text{fill}} \cdot j_{\text{Co}} = 100 \cdot 0.5 \cdot 5 = 250 \text{ Aturns}$$

As $n_c = 25$; $I = 250/(2 \cdot 25) = 5 \text{ A (RMS)}$. The wire gauge d_{Co} is:

$$d_{\text{Co}} = \sqrt{\frac{4}{\pi} \frac{I}{j_{\text{Co}}}} = \sqrt{\frac{4}{\pi} \frac{5}{5}} = 1.128 \text{ mm}$$

The pole pitch τ is

$$\tau = \frac{\pi D}{2p_1} = \frac{\pi \cdot 0.15}{2 \cdot 2} = 0.11775 \text{ m}$$

From (4.10)

$$K_{q1} = \frac{\sin \frac{\pi}{6}}{2 \sin \frac{\pi}{6 \cdot 2}} = 0.9659$$

$$K_{y1} = \sin \frac{\pi}{2} \cdot \frac{5}{6} = 0.966; \quad K_{w1} = K_{q1} K_{y1} = 0.9659 \cdot 0.966 = 0.933$$

The mmf fundamental amplitude, (from 4.14), is

$$W_1 = 2p_1 q n_c = 2 \cdot 2 \cdot 2 \cdot 25 = 200 \text{ turns / phase}$$

$$F_{1m} = \frac{3W_1 I \sqrt{2} K_{w1}}{\pi p_1} = \frac{3 \cdot 200 \cdot 5 \sqrt{2} \cdot 0.933}{\pi \cdot 2} = 628 \text{ At turns / pole}$$

From (4.16) the current sheet (loading) A_{1m} is,

$$A_{1m} = F_{1m} \frac{\pi}{\tau} = 628 \cdot \frac{\pi}{0.15} = 13155.3 \text{ At turns / m}$$

From (4.18),

$$K_{q7} = \frac{\sin(7\pi/6)}{2 \sin(7\pi/6 \cdot 2)} = -0.2588; \quad K_{y7} = \sin \frac{7\pi}{6} \cdot \frac{5}{6} = 0.2588$$

$$K_{w7} = -0.2588 \cdot 0.2588 = -0.066987$$

From (4.18),

$$F_{7m} = \frac{3W_1 I \sqrt{2} K_{q7} K_{y7}}{\pi p_1 7} = \frac{3 \cdot 200 \cdot 5 \sqrt{2} \cdot 0.066987}{\pi \cdot 2 \cdot 7} = 6.445 \text{ At turns / pole}$$

This is less than 1% of the fundamental $F_{1m} = 628 \text{ At turns / pole}$.

It may be shown that for 120° phase belts [10], the distribution (spread) factor K_{qv} is

$$K_{qv} = \frac{\sin v(\pi/3)}{q \sin(v\pi/3 \cdot q)} \quad (4.24)$$

For the same case $q = 2$ and $v = 1$, we find $K_{q1} = \sin \pi/3 = 0.867$. This is much smaller than 0.9659, the value obtained for the 60° phase belt, which explains in part why the latter case is preferred in practice.

Now that we introduced ourselves to a.c. windings through two case studies, let us proceed and develop general rules to design practical a.c. windings.

4.6. RULES FOR DESIGNING PRACTICAL A.C. WINDINGS

The a.c. windings for induction motors are usually built in one or two layers.

The basic structural element is represented by coils. We already pointed out (Figure 4.2) that there may be lap and wave coils. This is the case for single turn (bar) coils. Such coils are made of continuous bars (Figure 4.6a) for open slots

or from semibars bent and welded together after insertion in semiclosed slots (figure 4.6b).

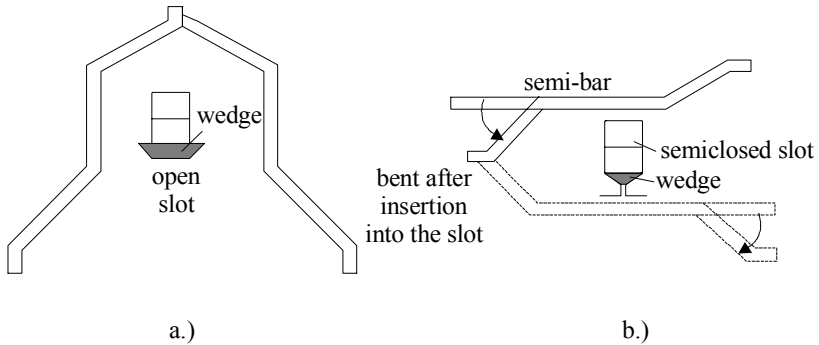


Figure 4.6 Bar coils: a.) continuous bar, b.) semi bar

These are preformed coils generally suitable for large machines.

Continuous bar coils may also be made from a few elementary conductors in parallel to reduce the skin effect to acceptable levels.

On the other hand, round-wire, mechanically flexible coils forced into semiclosed slots are typical for low power induction machines.

Such coils may have various shapes such as shown in Figure 4.7.

A few remarks are in order.

- Wire-coils for single layer windings, typical for low power induction motors (kW range and $2p_1 = 2\text{pole}$) have in general wave-shape;
- Coils for single layer windings are always full pitch as an average
- The coils may be concentrated or identical
- The main concern should be to produce equal resistance and leakage inductance per phase
- From this point of view, rounded concentrated or chain-shape identical coils are to be preferred for single layer windings

Double-layer winding coils for low power induction machines are of trapezoidal shape and round shape wire type (Figure 4.8a, b).

For large power motors, preformed multibar (rectangular wire) (Figure 4.8c) or unibar coils (Figure 4.6) are used.

Now to return to the basic rules for a.c. windings design let us first remember that they may be integer q or fractional q ($q = a+b/c$) windings with the total number of slots $N_s = 2p_1 q m$. The number of slots per pole could be only an integer. Consequently, for a fractional q , the latter is different and integer for a phase under different poles. Only the average q is fractional. Single-layer windings are built only with an integer q .

As one coil sides occupy 2 slots, it means that $N_s/2m = \text{an integer}$ (m —number of phases; $m = 3$ in our case) for single-layer windings. The number of inward current coil sides is evidently equal to the number of outward current coil sides.

For two-layer windings the allocation of slots per phase is performed in one (say, upper) layer. The second layer is occupied “automatically” by observing the coil pitch whose first side is in one layer and the second one in the second layer. In this case it is sufficient to have $N_s/m = \text{an integer}$.

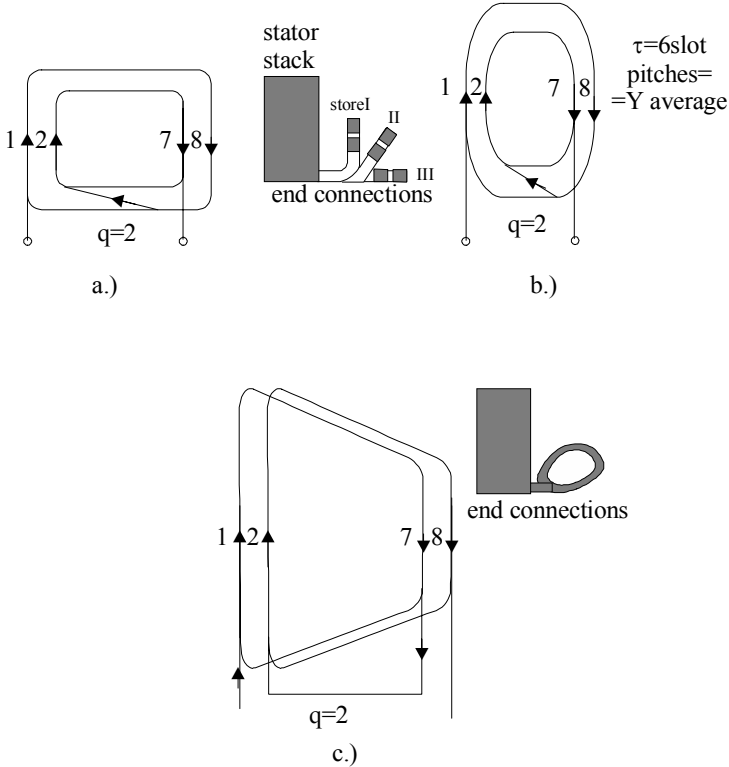


Figure 4.7 Full pitch coil groups/phase/pole—for $q = 2$ —for single layer a.c. windings:
a.) with concentrated rectangular shape coils and 2 (3) store end connections;
b.) with concentrated rounded coils; c.) with chain shape coils.

A pure traveling stator mmf (4.13), with an open rotor winding and a constant airgap (slot opening effects are neglected), when the stator and iron core permeability is infinite, will produce a no-load ideal flux density in the airgap as

$$B_{g10}(x, t) = \frac{\mu_0 F_{1m}}{g} \cos\left(\frac{\pi}{\tau} x - \omega_1 t\right) \quad (4.25)$$

according to Biot – Savart law.

This flux density will self-induce sinusoidal emfs in the stator windings.

The emf induced in coil sides placed in neighboring slots are thus phase shifted by α_{es}

$$\alpha_{es} = \frac{2\pi p_1}{N_s} \quad (4.26)$$

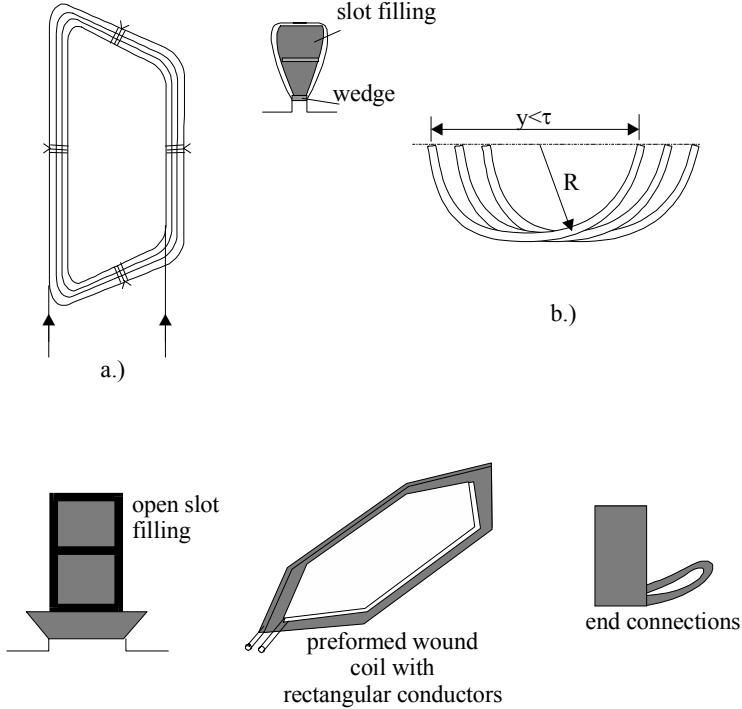


Figure 4.8 Typical coils for two-layer a.c. windings:

- a. trapezoidal flexible coil (round wire);
- b. rounded flexible coil (rounded wire);
- c. preformed wound coil (of rectangular wire) for open slots.

The number of slots with emfs in phase, t , is

$$t = \text{greatest common divisor } (N_s, p_1) = \text{g.c.d. } (N_s, p_1) \leq p_1 \quad (4.27)$$

Thus the number of slots with emfs of distinct phase is N_s/t . Finally the phase shift between neighboring distinct slot emfs α_{et} is

$$\alpha_{et} = \frac{2\pi t}{N_s} \quad (4.28)$$

If $\alpha_{es} = \alpha_{et}$, that is $t = p_1$, the counting of slots in the emf phasor star diagram is the real one in the machine.

Now consider the case of a single winding with $N_s = 24$, $2p_1 = 4$. In this case

$$\alpha_{es} = \frac{2\pi p_1}{N_s} = \frac{2\pi \cdot 2}{24} = \frac{\pi}{6} \quad (4.29)$$

$$t = \text{g.c.d.}(N_s, p_1) = \text{g.c.d.}(24, 2) = 2 = p_1 \quad (4.30)$$

So the number of distinct emfs in slots is $N_s/t = 24/2 = 12$ and their phase shift $\alpha_{et} = \alpha_{es} = \pi/6$. So their counting (order) is the natural one (Figure 4.9).

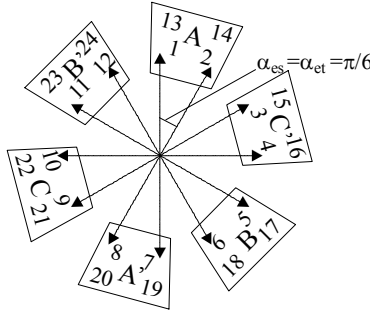


Figure 4.9 The star of slot emf phasors for a single-layer winding with $q = 2$, $2p_1 = 3$, $m = 3$, $N_s = 24$ slots.

The allocation of slots to phases to produce a symmetric winding is to be done as follows for

➤ single-layer windings

- Build up the slot emf phasor star based on calculating α_{et} , α_{es} , N_s/t distinct arrows counting them in natural order after α_{es} .
- Choose randomly $N_s/2m$ successive arrows to make up the inward current slots of phase A (Figure 4.9).
- The outward current arrows of phase A are phase shifted by π radians with respect to the inward current ones.
- By skipping $N_s/2m$ slots from phase A, we find the slots of phase B.
- Skipping further $N_s/2m$ slots from phase B we find the slots of phase C.

➤ double-layer windings

- Build up the slot emf phasor star as for single-layer windings.
- Choose N_s/m arrows for each phase and divide them into two groups (one for inward current sides and one for outward current sides) such that they are as opposite as possible.
- The same routine is repeated for the other phases providing a phase shift of $2\pi/3$ radians between phases.

It is well understood that the above rules are also valid for the case of fractional q . Fractional q windings are built only in two-layers and small q , to reduce the order of first slot harmonic.

➤ Placing the coils in slots

For single-layer, full pitch windings, the inward and outward side coil occupy entirely the allocated slots from left to right for each phase. There will be $N_s/2m$ coils/phase.

The chorded coils of double-layer windings, with a pitch y ($2\tau/3 \leq y < \tau$ for integer q and single pole count windings) are placed from left to right for each phase, with one side in one layer and the other side in the second layer. They are connected observing the inward (A, B, C) and outward (A', B', C') directions of currents in their sides.

➤ Connecting the coils per phase

The $N_s/2m$ coils per phase for single-layer windings and the N_s/m coils per phase for double-layer windings are connected in series (or series/parallel) such that for the first layer the inward/outward directions are observed. With all coils/phase in series, we obtain a single current path ($a = 1$). We may obtain “a” current paths if the coils from $2p_1/a$ poles are connected in series and, then, the “a” chains in parallel.

Example 4.2. Let us design a single-layer winding with $2p_1 = 2$ poles, $q = 4$, $m = 3$ phases.

Solution

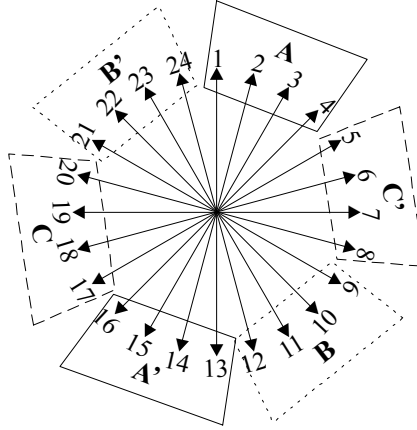


Figure 4.10 The star of slot emf phasors for a single-layer winding: $q = 1$, $2p_1 = 2$, $m = 3$, $N_s = 24$.

The angle α_{cs} (4.26), t (4.27), α_{et} (4.28) are

$$N_s = 2p_1 q m = 24; \quad \alpha_{cs} = \frac{2\pi P_1}{N_s} = \frac{2\pi \cdot 1}{24} = \frac{\pi}{12}$$

$$t = \text{g.c.d.}(N_s, P_1) = \text{g.c.d.}(24, 1) = 1$$

$$\alpha_{et} = \frac{2\pi t}{N_s} = \frac{2\pi \cdot 1}{24} = \frac{\pi}{12}$$

Also the count of distinct arrows of slot emf star $N_s/t = 24/1 = 24$.

Consequently the number of arrows in the slot emf star is 24 and their order is the real (geometrical) one (1, 2, ... 24)–Figure 4.10.

Making use of Figure 4.10, we may thus allocate the slots to phases as in Figure 4.11.

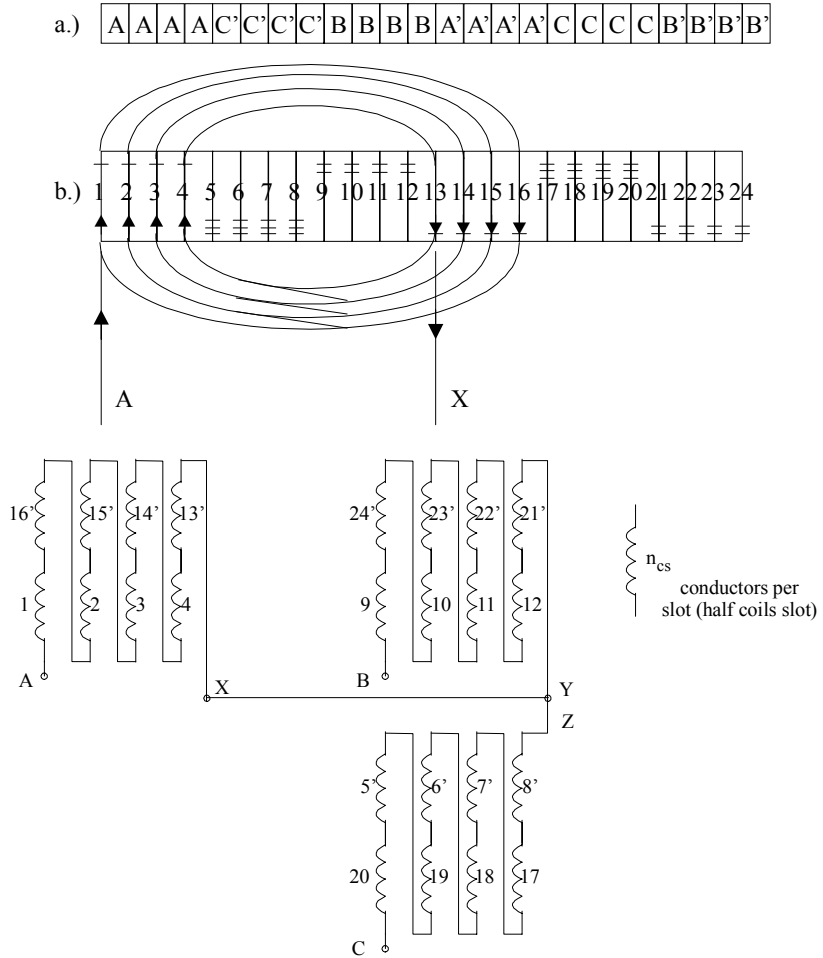


Figure 4.11 Single-layer winding layout
 a.) slot/phase allocation; b.) rounded coils of phase A; c.) coils per phase

Example 4.3. Let us consider a double-layer three-phase winding with $q = 3$, $2p_1 = 4$, $m = 3$, ($N_s = 2p_1qm = 36$ slots), chorded coils $y/\tau = 7/9$ with $a = 2$ current paths.

Solution

Proceeding as explained above, we may calculate α_{es} , t , α_{et} :

$$\alpha_{es} = \frac{2\pi P_1}{N_s} = \frac{2\pi \cdot 2}{36} = \frac{\pi}{9}$$

$$t = \text{g.c.d.}(36, 2) = 2$$

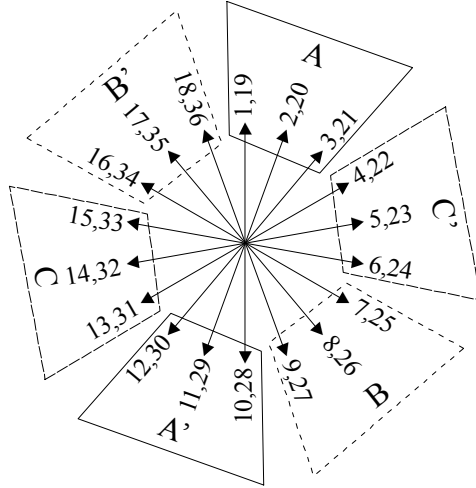


Figure 4.12 The star of slot emf phasors for a double-layer winding (one layer shown) with $2p_1 = 4$ poles,
 $q = 3$ slots/pole/phase, $m = 3$, $N_s = 36$

$$\alpha_{et} = \frac{2\pi t}{N_s} = \frac{2\pi \cdot 2}{36} = \frac{\pi}{9} = \alpha_{es} \quad N_s / t = 36 / 2 = 18$$

There are 18 distinct arrows in the slot emf star as shown in [Figure 4.12](#).

The winding layout is shown in [Figure 4.13](#). We should notice the second layer slot allocation lagging by $\tau - y = 9 - 7 = 2$ slots, the first layer allocation.

Phase A produces 4 fully symmetric poles. Also, the current paths are fully symmetric. Equipotential points of two current paths $U - U'$, $V - V'$, $W - W'$ could be connected to each other to handle circulating currents due to, say, rotor eccentricity.

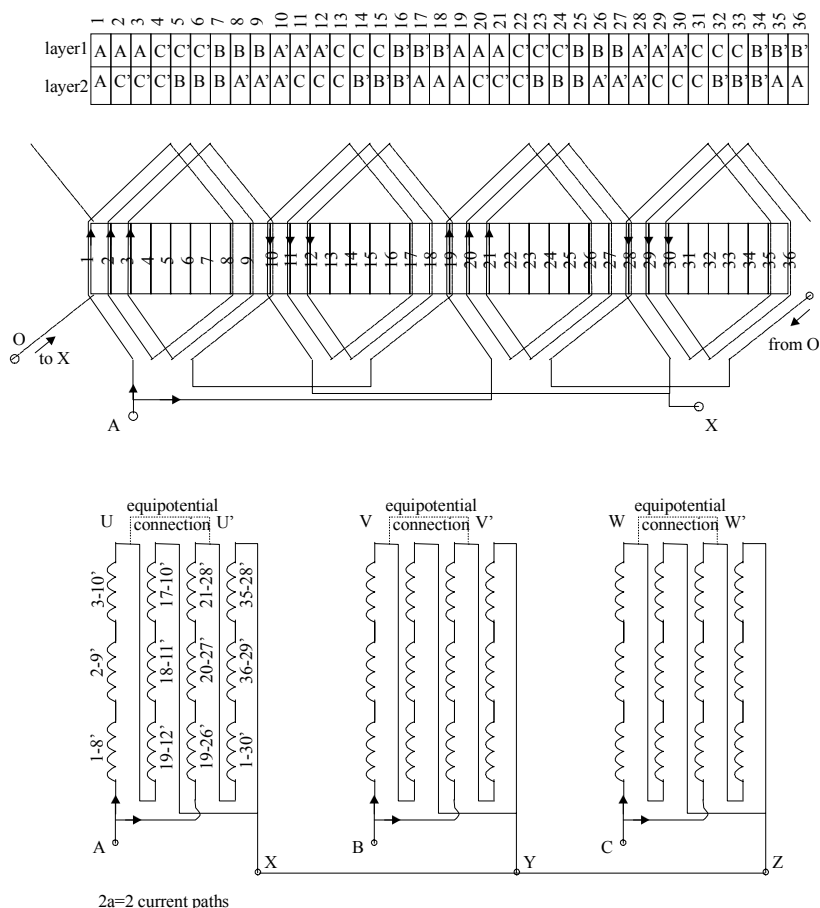


Figure 4.13 Double-layer winding: $2p_1 = 4$ poles, $q = 3$, $y/\tau = 7/9$, $N_s = 36$ slots, $a = 2$ current paths.

Having two current paths, the current in the coils is half the current at the terminals. Consequently, the wire gauge of conductors in the coils is smaller and thus the coils are more flexible and easier to handle.

Note that using wave coils is justified in single-bar coils to reduce the external leads to one by which the coils are connected to each other in series. Copper, labor, and space savings are the advantages of this solution.

4.7. BASIC FRACTIONAL q THREE-PHASE A.C. WINDINGS

Fractional q a.c. windings are not typical for induction motors due to their inherent pole asymmetry as slot/phase allocation under adjacent poles is not the same in contrast to integer q three-phase windings. However, with a small q ($q \leq$

3) to reduce the harmonics content of airgap flux density, by increasing the order of the first slot harmonic from $6q \pm 1$ for integer q to $6(ac + b) \pm 1$ for $q = (ca + b)/c = \text{fractional}$ two-layer such windings are favoured to single-layer versions. To set the rules to design such a standard winding—with identical coils—we proceed with an example.

Let us consider a small induction motor with $2p_1 = 8$ and $q = 3/2$, $m = 3$. The total number of slots $N_s = 2p_1 q m = 2 \cdot 4 \cdot 3/2 \cdot 3 = 36$ slots. The coil span y is

$$y = \text{integer}(N_s/2p_1) = \text{integer}(36/8) = 4 \text{ slot pitches} \quad (4.31)$$

The parameters t , α_{es} , α_{et} are

$$t = \text{g.c.d.}(N_s, p_1) = \text{g.c.d.}(36, 4) = 4 = p_1 \quad (4.33)$$

$$\alpha_{es} = \frac{2\pi p_1}{N_s} = \frac{\pi \cdot 8}{36} = \frac{2\pi}{9} = \alpha_{et} \quad (4.34)$$

The count of distinct arrows in the star of slot emf phasors is $N_s/t = 36/4 = 9$. This shows that the slot/phase allocation repeats itself after each pole pair (for an integer q it repeats after each pole). Thus mmf subharmonics, or fractional space harmonics, are still absent in this case of fractional q . This property holds for any $q = (2l + 1)/2$ for two-layer configurations.

The star of slot emf phasors has q arrows and the counting of them is the natural one ($\alpha_{es} = \alpha_{et}$) (Figure 4.14a).

A few remarks in Figure 4.14 are in order

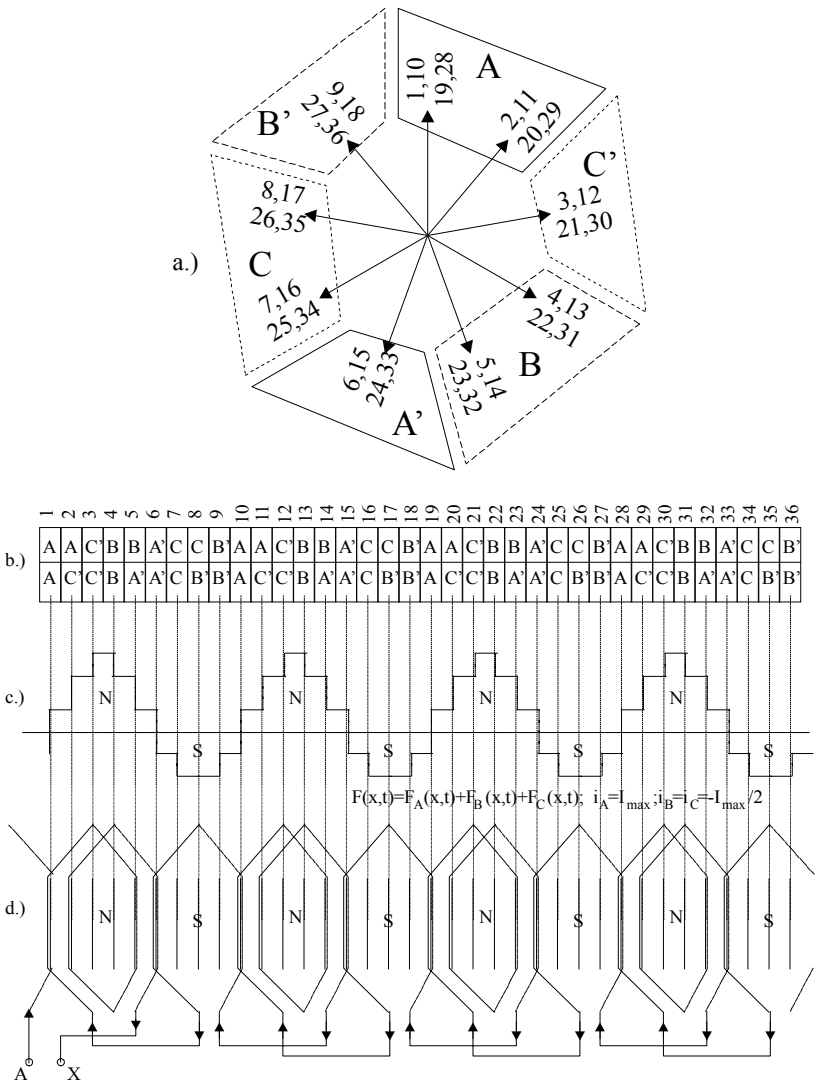
- The actual value of q for each phase under neighboring poles is 2 and 1, respectively, to give an average of $3/2$
- Due to the periodicity of two poles (2τ), the mmf distribution does not show fractional harmonics ($v < 1$)
- There are both odd and even harmonics, as the positive and negative polarities of mmf (Figure 4.14c) are not fully symmetric
- Due to a two pole periodicity we may have $a = 1$ (Figure 4.14d), or $a = 2, 4$
- The chording and distribution (spread) factors (K_{y1} , K_{q1}) for the fundamental may be determined from Figure 4.14e using simple phasor composition operations.

$$K_{y1} = \sin \left[\frac{\pi p_1}{N_s} \text{integer}(N_s / 2p_1) \right] \quad (4.35)$$

$$K_{q1} = \frac{1 + 2 \cos \left(\frac{\pi t}{N_s} \right)}{3} \quad (4.36)$$

This is a kind of general method valid both for integer and fractional q .

Extracting the fundamental and the space harmonics of the mmf distribution (Figure 4.14c) takes implicit care of these factors both for the fundamental and for the harmonics.



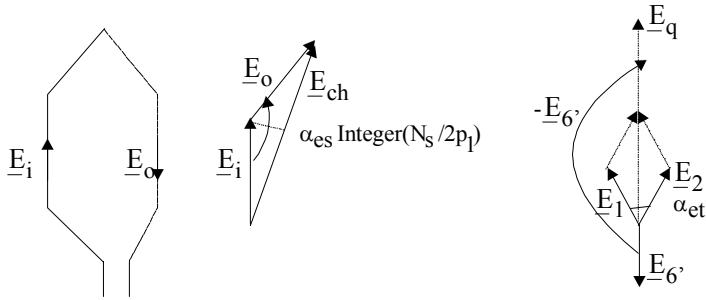


Figure 4.14 Fractionary q ($q = 3/2$, $2p_1 = 8$, $m = 3$, $N_s = 36$) winding
a.) emf star, b.) slot/phase allocation, c.) mmf,
d.) coils of phase A, e.) chording and spread factors

4.8. BASIC POLE-CHANGING THREE-PHASE A.C. WINDINGS

From (4.20) the speed of the mmf fundamental dx/dt is

$$\left(\frac{dx}{dt} \right)_{v=1} = 2\pi f_1 \quad (4.37)$$

The corresponding angular speed is

$$\Omega_1 = \frac{dx}{dt} \frac{2}{D} = \frac{2\pi f_1}{p_1}; \quad n_1 = \frac{f_1}{p_1} \quad (4.38)$$

The mmf fundamental wave travels at a speed $n_1 = f_1/p_1$. This is the ideal speed of the motor with a cage rotor.

Changing the speed may be accomplished either by changing the frequency (through a static power converter) or by changing the number of poles.

Changing the number of poles to produce a two-speed motor is a traditional method. Its appeal is still strong today due to low hardware costs where continuous speed variation is not required. In any case, the rotor should have a squirrel cage to accommodate both pole pitches. Even in variable speed drives with variable frequency static converters, when a very large constant power speed range (over 2(3) to 1) is required, such a solution should be considered to avoid a notable increase in motor weight (and cost).

Two – speed induction generators are also used for wind energy conversion to allow a notable speed variation to extract more energy from the wind speed.

There are two possibilities to produce a two-speed motor. The most obvious one is to place two distinct windings in the slots. The number of poles would be $2p_1 > 2p_2$. However the machine becomes very large and costly, while for the winding placed on the bottom of the slots the slot leakage inductance will be very large with all due consequences.

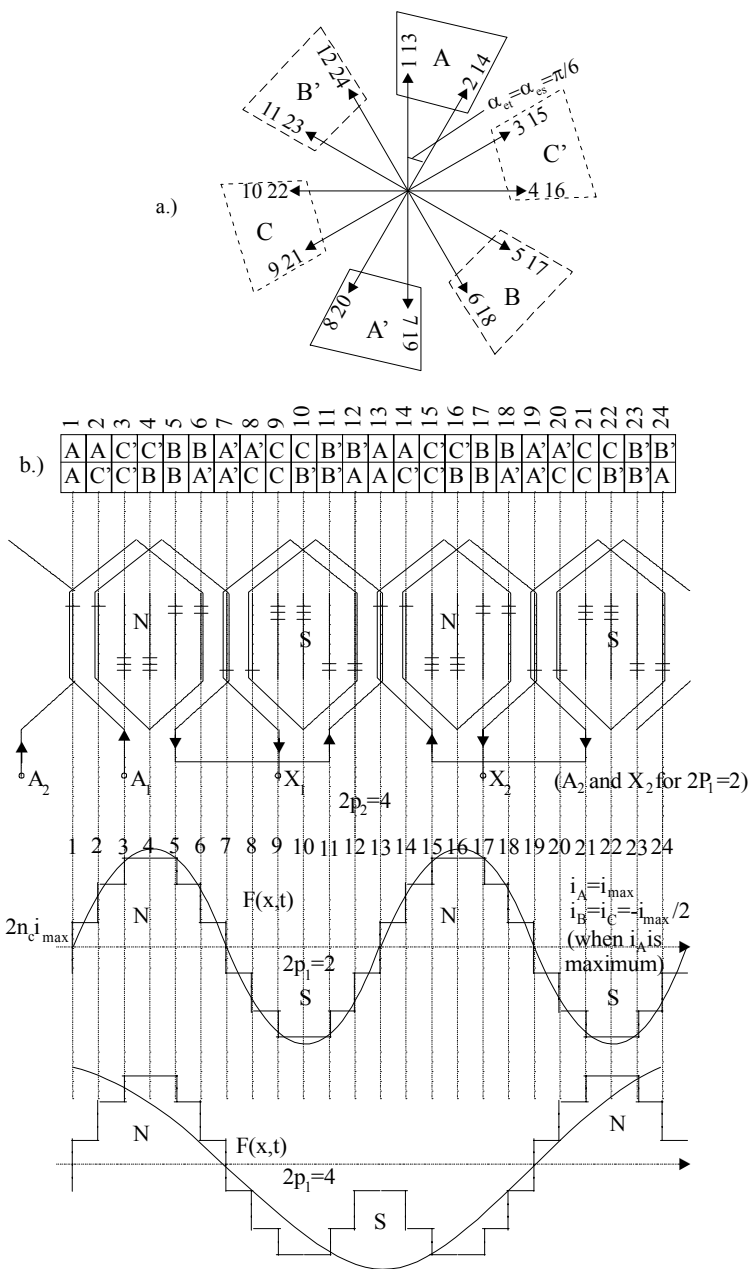


Figure 4.15 2/4 pole winding ($N_s = 24$)

a.) emf star, b.) slot/phase allocation, c.) coils of phase A, d.), e.) mmf for $2p_2 = 4$ and $2p_1 = 2$

Using a pole-changing winding seems thus a more practical solution. However standard pole-changing windings have been produced mainly for $2p_1/2p_2 = 1/2$ or $2p_1/2p_2 = 1/3$.

The most acclaimed winding has been invented by Dahlander and bears his name.

In essence the current direction (polarity) in half of a $2p_2$ pole winding is changed to produce only $2p_1 = 2p_2/2$ poles. The two-phase halves may be reconnected in series or parallel and Y or Δ connections of phases is applied. Thus, for a given line voltage and frequency supply, with various such connections, constant power or constant torque or a certain ratio of powers for the two speeds may be obtained.

Let us now proceed with an example and consider a two-layer three-phase winding with $q = 2$, $2p_2 = 4$, $m = 3$, $N_s = 24$ slots, $y/\tau = 5/6$ and investigate the connection changes to switch it to a two pole ($2p_1 = 2$) machine.

The design of such a winding is shown on Figure 4.15. The variables are $t = \text{g.c.d}(N_s, p_2) = 2 = p_2$, $\alpha_{es} = \alpha_{et} = 2\pi p_2/N_s = \pi/6$, and $N_s/t = 12$. The star of slot emf phasors is shown in Figure 5.15a.

Figure 4.15c illustrates the fact that only the current direction in the section A2 $\rightarrow$ X2 of phase A is changed to produce a $2p_1 = 2$ pole winding. A similar operation is done for phases B and C.

A possible connection of phase halves called $\Delta/2Y$ is shown in Figure 4.16.

It may be demonstrated that for the Δ ($2p_2 = 4$)/ $2Y$ ($2p_1 = 2$) connection, the power obtained for the two speeds is about the same.

We also should notice that with chorded coils for $2p_2 = 4$ ($y/\tau = 5/6$), the mmf distribution for $2p_1 = 2$ has a rather small fundamental and is rich in harmonics.

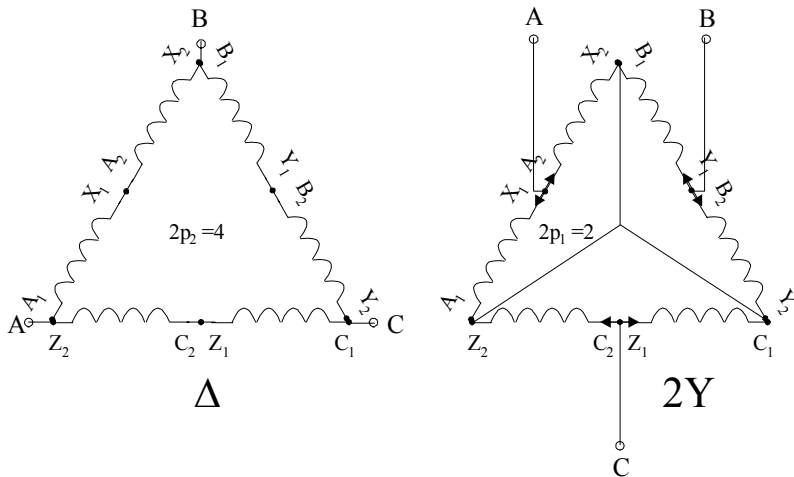


Figure 4.16 2/4 pole winding connection for constant power

In order to achieve about the same winding factor for both pole numbers, the coil span should be greater than the pole pitch for the large number of poles ($y = 7, 8$ slot pitches in our case). Even if close to each other, the two winding factors are, in general, below 0.85.

The machine is thus larger than usual for the same power and care must be exercised to maintain the acceptable noise level.

Various connections of phases may produce designs with constant torque (same rated torque for both speeds) or variable torque. For example, a Y – YY parallel connection for $2p_2/2p_1 = 2/1$ is producing a ratio of power $P_2/P_1 = 0.35 - 0.4$ as needed for fan driving. Also, in general, when switching the pole number we may need modify the phase sequence to keep the same direction of rotation.

One may check if this operation is necessary by representing the stator mmf for the two cases at two instants in time. If the positive maximum of the mmfs advances in time in opposite directions, then the phase sequence has to be changed.

4.9. TWO-PHASE A.C. WINDINGS

When only single phase supply is available, two-phase windings are used. One is called the main winding (M) and the other, connected in series with a capacitor, is called the auxiliary winding (Aux).

The two windings are displaced from each other, as shown earlier in this chapter, by 90° (electrical), and are symmetrized for a certain speed (slip) by choosing the correct value of the capacitance. Symmetrization for start (slip = 0) with a capacitance C_{start} and again for rated slip with a capacitance C_{run} is typical ($C_{start} \gg C_{run}$).

When a single capacitor is used, as a compromise between acceptable starting and running, the two windings may be shifted in space by more than 90° (105° to 110°) (Figure 4.17).

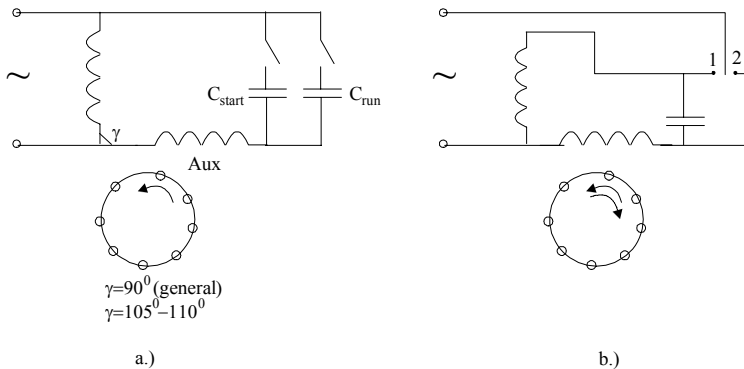


Figure 4.17 Two-phase induction motor

a.) for unidirectional motion,

b.) for bidirectional motion (1 - closed for forward motion; 2 - closed for backward motion)

Single capacitor configurations for good start are characterized by the disconnection of the auxiliary winding (and capacitor) after starting. In this case the machine is called capacitor start and the main winding occupies 66% of stator periphery. On the other hand, if bi-directional motion is required, the two windings should each occupy 50% of the stator periphery and should be identical.

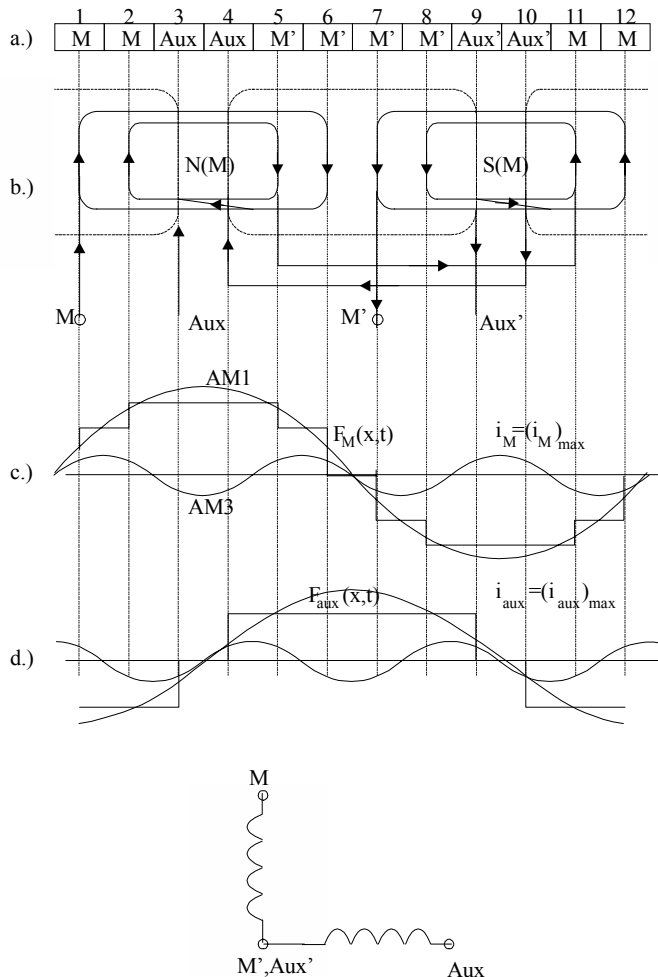


Figure 4.18 Capacitor start motor two-phase single-layer winding; $2p_1 = 2$ poles, $N_s = 12$ slots
a.) slot/phase allocation: $M/Aux = 2/1$, b.) coils per phase connections,
c.) mmf distribution for main phase (M), d.) mmf distribution for auxiliary phase (Aux)

Two-phase windings are used for low power (0.3 to 2 kW) and thus have rather low efficiency. As these motors are made in large numbers due to their

use in home appliances, any improvement in the design may be implemented at competitive costs. For example, to reduce the mmf harmonics content, both phases may be placed in all (most) slots in different proportions. Notice that in two-phase windings multiple of three harmonics may exist and they may deteriorate performance notably.

Let us first consider the capacitor-start motor windings.

As the main winding (M) occupies 2/3 of all slots (Figure 4.18a, b), its mmf distribution (Figure 4.18c) is notably different from that of the auxiliary winding (Figure 4.18d). Also the distribution factors for the two phases (K_{q1M} and K_{q1Aux}) are expected to be different as $q_M = 4$ and $q_{Aux} = 2$.

For $N_s = 16$, $2p_1 = 2$, the above winding could be redesigned for reversible motion where both windings occupy the same number of slots. In that case the windings look like those shown in Figure 4.19.

The two windings have the same number of slots and same distribution factors and, have the same number of turns per coil and same wire gauge only for reversible motion when the capacitor is connected to either of the two phases. The quantity of copper is not the same since the end connections may have slightly different lengths for the two phases.

Double-layer windings, for this case ($q_M = q_{Aux} = \text{slot/pole/phase}$), may be built as done for three-phase windings with identical chorded coils. In this case, even for different number of turns/coil and wire gauge–capacitor run motors–the quantity of copper is the same and the ratio of resistances and self inductances is proportional to the number of turns/coil squared.

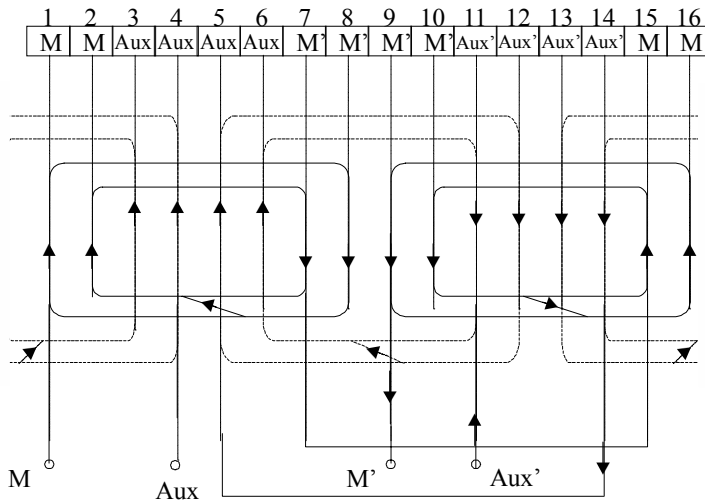


Figure 4.19 Reversible motion (capacitor run) two-phase winding with $2p_1 = 2$, $N_s = 16$, single-layer

As mentioned above and in Chapter 2, capacitor induction motors of high performance need almost sinusoidal mmf distribution to cancel the various bad space harmonic influences on performance.

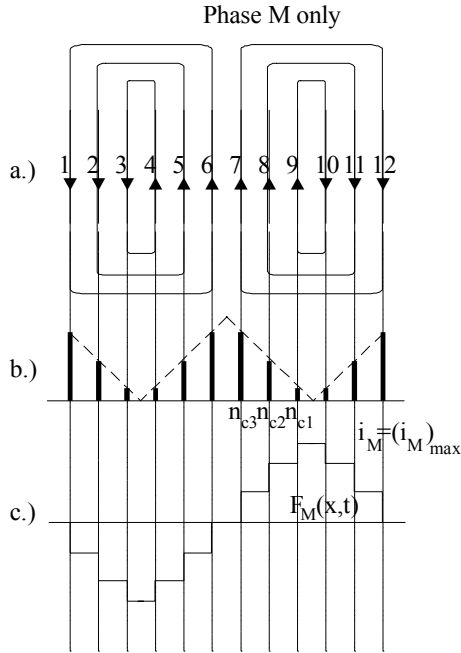


Figure 4.20. Quasi-sinusoidal winding (main winding M shown)
a.) coils in slot/one layer, b.) turns/coil/slot, c.) mmf distribution

One way to do it is to build almost sinusoidal windings; that is, all slots contain coils of both phases but in different proportions (number of turns) so as to obtain sinusoidal mmf. An example for the 2 pole 12 slot case is given in [Figure 4.20](#).

Now if the two windings are equally strong, having the same amount of copper, the slots could be identical. If not, in the case of a capacitor-start, only auxiliary winding, part of the slots may have a smaller area. This way, the motor weight may be reduced. It is evident from [Figure 4.20c](#) that the mmf distribution is now much closer to a sinusoid.

Satisfactory results may be obtained for small power resistor started induction motors if the distribution of conductor/slot/phase runs linear rather than sinusoidal with a flat (open) zone of one third of a pole pitch. In this case, however, the number of slots N_s should be a multiple of $6p_1$.

An example of a two one-layer sinusoidal winding is shown in [Figure 4.21](#), for $2p_1 = 2$, $N_s = 24$ slots.

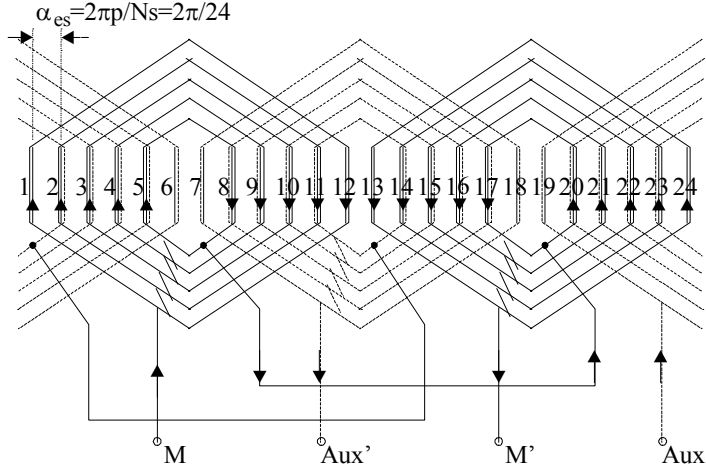


Figure 4.21 Two-pole two-phase winding with sinusoidal conductor count/slot/phase distribution

Similar to [Figure 4.20b](#), the number of turns/coils for the main winding in slots 1 to 5 is

$$\begin{aligned}
 n_{c1}^m &= K_M \cos \alpha_{es} / 2 = n_{c12}^m = n_{c13}^m = n_{c24}^m = 0.991 \cdot K_M \\
 n_{c2}^m &= K_M \cos \frac{3}{2} \alpha_{es} / 2 = n_{c11}^m = n_{c14}^m = n_{c23}^m = 0.924 \cdot K_M \\
 n_{c3}^m &= K_M \cos \frac{5}{2} \alpha_{es} / 2 = n_{c10}^m = n_{c15}^m = n_{c22}^m = 0.7933 \cdot K_M \\
 n_{c4}^m &= K_M \cos \frac{7}{2} \alpha_{es} / 2 = n_{c9}^m = n_{c16}^m = n_{c21}^m = 0.6087 \cdot K_M \\
 n_{c5}^m &= K_M \cos \frac{9}{2} \alpha_{es} / 2 = n_{c8}^m = n_{c17}^m = n_{c20}^m = 0.38268 \cdot K_M
 \end{aligned} \tag{4.39}$$

A similar division of conductor counts is valid for the auxiliary winding.

$$\begin{aligned}
 n_{c6}^a &= K_A \cos \alpha_{es} / 2 = n_{c7}^a = n_{c18}^a = n_{c19}^a = 0.991 \cdot K_A \\
 n_{c5}^a &= K_A \cos \frac{3}{2} \alpha_{es} / 2 = n_{c8}^a = n_{c17}^a = n_{c20}^a = 0.924 \cdot K_A \\
 n_{c4}^a &= K_A \cos \frac{5}{2} \alpha_{es} / 2 = n_{c9}^a = n_{c16}^a = n_{c21}^a = 0.7933 \cdot K_A \\
 n_{c3}^a &= K_A \cos \frac{7}{2} \alpha_{es} / 2 = n_{c10}^a = n_{c15}^a = n_{c22}^a = 0.6087 \cdot K_A \\
 n_{c2}^a &= K_A \cos \frac{9}{2} \alpha_{es} / 2 = n_{c11}^a = n_{c14}^a = n_{c23}^a = 0.38268 \cdot K_A
 \end{aligned} \tag{4.40}$$

4.10. POLE-CHANGING WITH SINGLE-PHASE SUPPLY INDUCTION MOTORS

A $2/1 = 2p_2/2p_1$ pole-changing winding for single-phase supply can be approached with a three-phase pole-changing winding. The Dahlander winding may be used (see Figure 4.16) as in Figure 4.22.

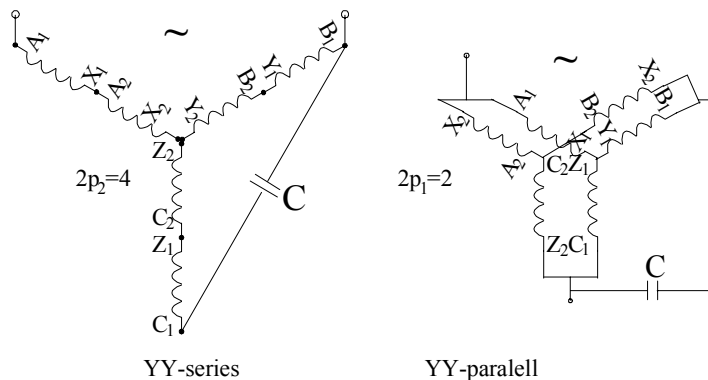


Figure 4.22 4/2 pole two-phase winding for single-phase supply made from a three-phase winding

4.11. SPECIAL TOPICS ON A.C. WINDINGS

Most of the windings treated so far in this chapter are in industrial use today. At the same time, new knowledge has surfaced recently. Here we review the most significant part of it.

➤ A new general formula for mmf distribution

Let us consider the slot opening angle β , n_{cK} —conductors/slot/phase, $i_K(t)$ current in slot K, θ_K —slot position per periphery.

A linear current density in a slot $I_K(\theta_K, t)$ may be defined as

$$\begin{cases} I_K(\theta, t) = \frac{2n_{cK} \cdot i_K(t)}{\beta D} & \text{for } \theta \in (\theta_K - \beta/2, \theta_K + \beta/2) \\ I_K(\theta, t) = 0 & \text{for } \theta \notin (\theta_K - \beta/2, \theta_K + \beta/2) \end{cases} \quad (4.41)$$

For a single slot, the linear current density may be decomposed in mechanical harmonics as

$$I_K(\theta, t) = \frac{2n_{cK} \cdot i_K(t)}{\pi D} \left[\frac{1}{2} + \sum_v K_v \cos(\theta - \theta_v) \right] \quad (4.42)$$

with

$$K_v = \frac{2}{v\beta} \sin \frac{v\beta}{2} \quad (4.43)$$

The number n_{cK} may be positive or negative depending on inward/outward sense of connections and it is zero if no conductors belong to that particular phase. By adding the contributions of all slots per phase A, we obtain its mmf [10,11].

$$F_A(\theta, t) = \frac{2}{\pi} W_1 i_A(t) \sum_v^{\infty} \frac{K_v K_{wv}}{v} \sin(\theta - \theta_{Av}) \quad (4.44)$$

$$\text{with} \quad K_{wv} = \frac{1}{2W_1} \sqrt{\left(\sum_{K=1}^{N_s} n_{cK} \cos v\theta_K \right)^2 + \left(\sum_{K=1}^{N_s} n_{cK} \sin v\theta_K \right)^2} \quad (4.45)$$

$$\theta_{Av} = \frac{1}{v} \tan^{-1} \left[\frac{\left(\sum_{K=1}^{N_s} n_{cK} \sin v\theta_K \right)}{\left(\sum_{K=1}^{N_s} n_{cK} \cos v\theta_K \right)} \right] \quad (4.46)$$

where W_1 is the total number of turns/phase. When there is more than one current path ($a > 1$), W_1 is replaced by W_{1a} (turns/phase/path) and only the slots/phase/path are considered.

K_{wv} is the total winding factor: spread and chording factors multiplied. This formula is valid for the general case (integer q or fractional q windings, included). The effect of slot opening on the phase mmf is considered through the factor $K_v < 1$ (which is equal to unity for zero slot opening – ideal case). In general, in an ideal case, the mmf harmonics amplitude is reduced by slot opening, but so is its fundamental.

➤ **Low space harmonic content windings**

Fractional pitch windings have been, in general, avoided for induction motor, especially due to low order (sub or fractional) harmonics and their consequences: parasitic torques, noise, vibration, and losses.

However, as basic resources for IM optimization are exhausted, new ways to reduce copper weight and losses are investigated. Among them, fractional windings with two pole symmetry ($q = (2l + 1)/2$) are investigated thoroughly [9,12]. They are characterised by the absence of sub (fractional) harmonics. The case of $q = 3/2$ is interesting low power motors for low-speed high-pole-count induction generators and motors. In essence, the number of turns of the coils in the two groups/phase belonging to neighboring poles (one with $q_1 = 1$ and the other with $q_2 = 1 + 1$) varies so that designated harmonics could reduce or destroy. Also, using coils of a different number of conductors in various slots of a phase with standard windings (say $q = 2$) should lead to the cancellation of low order mmf harmonics and thus render the motors less noisy.

A reduction of copper weight and (or) an increase in efficiency may thus be obtained. The price is using nonidentical coils.

The general formula of the winding factor (4.45) may be simplified for the case of $q = 3/2, 5/2, 7/2 \dots$ where two-pole symmetry is secured and thus the summation in (4.45) extends along only two poles.

With γ_i the coil pitch, the winding factor K_{wv} , for two poles, is [9]

$$K_{wv} = \sum_i^{q_1=1} n_{ci}' \sin \frac{v\gamma_i}{2} - (-1)^v \sum_K^{q_2=1+1} n_{cK}' \sin \frac{v\gamma_K}{2} \quad (4.47)$$

n_{ci}' and n_{cK}' are relative numbers of turns with respect to total turns/polepair/phase; therefore the coil pitches may be different under the two poles. Also, the number of such coils for the first pole is $q_1 = 1$ and for the second is $q_2 = 1 + 1$ ($q = (2l + 1)/2$).

As $q = (2l + 1)/2$, the coil pitch angles γ_i will refer to an odd number of slot pitches while γ_K refers to an even number of slot pitches. So it could be shown that the winding factor K_{wv} of some odd and even harmonics pairs are equal to each other. For example, for $q = 3/2$, $K_{w4} = K_{w5}$, $K_{w2} = K_{w7}$, $K_{w1} = K_{w8}$. These relationships may be used when designing the windings (choosing γ_i , γ_K and n_{ci}' , n_{cK}') to cancel some harmonics.

➤ A quasisinusoidal two-layer winding [8]

Consider a winding with $2l + 1$ coils/phase/pole pairs. Consequently, $q = (2l + 1)/2$ slot/pole/phase.

There are two groups of coils per phase. One for the first pole containing $K + 1$ coils of span $3l + 1, 3l - 1 \dots$ and, for the second pole, K coils of span $3l, 3l - 2$. The problem consists of building a winding in two-layers to cancel all harmonics except those in multiples of 3 (which will be inactive in star connection of phases anyway) and the $3(2l + 1) \pm 1$, or slot harmonics.

$$\sum K_{wv} (n_{ci}' + n_{cK}') = 0; v = 2, 4, 5, \dots (\text{in all } 2l \text{ values}) \quad (4.48)$$

$$\text{with} \quad \sum_{i=1}^{2l+1} (n_{ci}' + n_{cK}') = 1 \quad (4.49)$$

This linear system has a unique solution, which, in the order of increased number of turns, yields

$$n_{ci}' = 4 \sin \left(\frac{\pi}{6(2l+1)} \right) \sin \frac{(2i-1)\pi}{6(2l+1)}; i = 1, 2, 3 \dots 2l+1 \quad (4.50)$$

For the fundamental mmf wave, the winding factor K_{w1} is

$$K_{w1} = (2l+1) \sqrt{3} \sin \frac{\pi}{6(2l+1)} \quad (4.51)$$

For $l = 1$, $2l + 1 = 3$ coils/phase/pole pair. In this case the relative number of turns per coil for phase A are (from 4.50): $n_{c1}' = 0.1206$, $n_{c2}' = 0.3473$, $n_{c3}' = 0.5321$.

As we can see, $n_{c1}' + n_{c3}' \cong 2 n_{c2}'$ so the filling of all slots is rather uniform though not identical.

The winding layout is shown in [Figure 4.23](#).

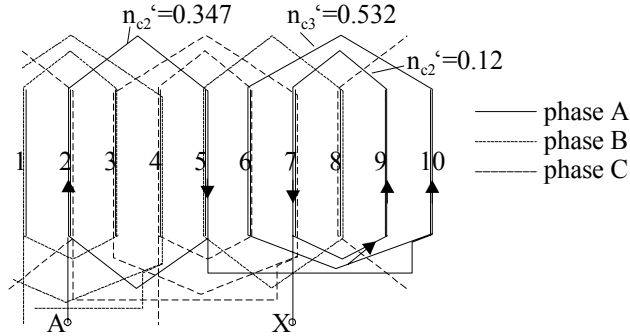


Figure 4.23 Sinusoidal winding with $q = 3/2$ in two-layers

The fundamental winding factor of this winding is $K_{w1} = 0.9023$, which is satisfactory (both distribution and chording factors are included). To further cancel the multiples of 3 (3l) harmonics, two/three layer windings based on the same principle have been successfully tested [9] though their fundamental winding factor is below 0.8.

The same methodology could be applied for $q = \text{integer}$, by using concentrated coils with various numbers of turns to reduce mmf harmonics. Very smooth operation has been obtained with such a 2.2 kW motor with $q = 2$, $2p_1 = 4$, $N_s = 24$ slots up to 6000 rpm. [12]

In general, for the same copper weight, such sinusoidal windings are claimed to produce a 20% increase in starting torque and 5 to 8dB noise reduction, at about the same efficiency.

➤ Better pole-changing windings

Pole amplitude modulation [3–5] and symmetrization [13–14] techniques have been introduced in the sixties and revisited in the nineties. [15] Like interspersing [10, pp.37–39], these methods have produced mixed results so far. On the other hand, the so called "3 equation principle" for better pole-changing winding has been presented in [7] for various pole count combinations. This is also a kind of symmetrization method but with a well defined methodology. The connections of phases is Δ/Δ and $(3Y + 2Y)/3Y$ for $2p_2$ and $2p_1$ pole counts, respectively. Higher fundamental winding factor, lower harmonics content, better winding and core utilisation, and simpler switching devices from $2p_2$ to $2p_1$ poles than with Dahlander connections are all merits of such a methodology. Two single-throw switches (soft starters) suffice ([Figure 4.24](#)) to change speed.

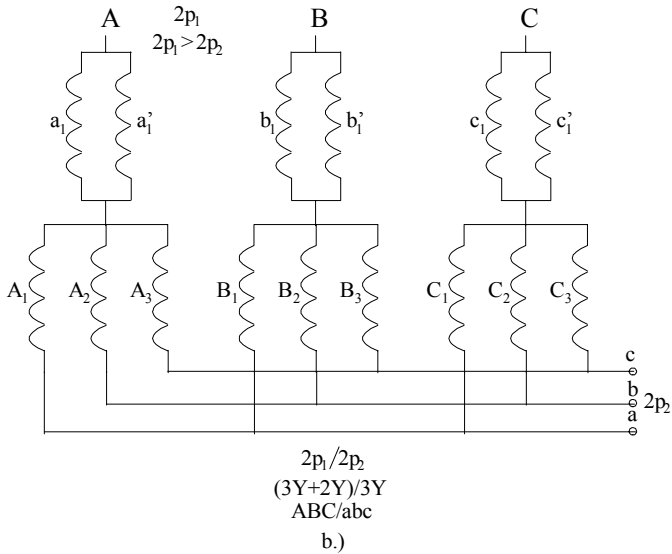
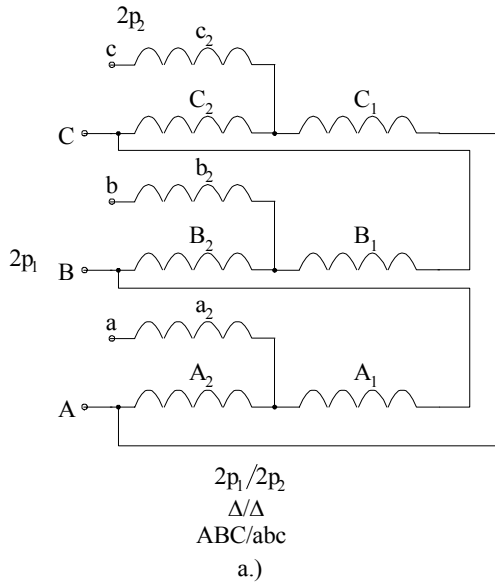


Figure 4.24 $2p_1/2p_2$ pole connections

a.) Δ/Δ ; b.) $(3Y + 2Y)/3Y$

The principle consists of dividing the slot periphery into three equivalent parts by three slots counted n , $n + N_s/3$, and $n + 2N_s/3$. Now the electrical angle between these slots mmf is $2\pi/3p_1$ as expected ($p_1 = p_1, p_2$). If $2p_1$ is not a

multiple of three the coils in slots n , $n + N_s/3$ and $n + 2N_s/3$ belong to all three-phases and the winding is to be symmetric. The Δ connection is used (Figure 4.24). In contrast, if $2p_1$ is a multiple of 3, the emfs in slots n , $n + N_s/3$, $n + N_s + 2N_s/3$ are in phase and thus belong to the same phase and we may build three parallel branches (paths) per phase in a 3Y connection (Figure 4.24b). In general, $n = 1, 2, \dots N_s/3$.

As already mentioned, the Δ/Δ connection (Figure 4.24a) is typical for nonmultiple of 3 pole counts ($2p_1/2p_2 = 8/10$, for example).

Suppose every coil group $A_{1,2,3}$, $B_{1,2,3}$, $C_{1,2,3}$ is made of t_1 coils ($t_1 < N_s/6$). The coil group A_1 is made of t_1 coils in slots $n_1, n_2, \dots, n_{t_1}$ while groups B_1 and C_1 are displaced by $N_s/3$ and $2N_s/3$, respectively. The sections A_1, B_1, C_1 , which belong to the three-phases, do not change phase when the number of poles is changed from $2p_1$ to $2p_2$.

On the other hand, coil groups A_2, B_2, C_2 are composed of $t_2 < N_s/6$ coils and thus coil group A_2 refers to slots $n_1', n_2', \dots, n_{t_2}'$ while coil groups B_2 and C_2 are displaced by $N_s/3$ and $2N_s/3$ with respect to group A_2 . Again, sections A_2, B_2, C_2 are symmetric.

It may be shown that for $2p_1$ and $2p_2$ equal to $2p_1 = 6K_1 + 2(4)$ and $2p_2 = 6K_2 + 1$, respectively, the mmf waves for the two pole counts travel in the same direction.

On the other hand, if $2p_1 = 6K_1 + 2(4)$ and $2p_2 = 6K_2 + 4(2)$, the mmf waves for the two pole counts move in opposite directions. Swapping two phases is required to keep the same direction of motion for both pole counts (speeds).

In general, t_1 or t_2 are equal to $N_s/6$. For voltage adjustment for higher pole counts, $t_3 = N_s/3 - t_1$ slots/phase are left out and distributed later, also with symmetry in mind.

Table 4.1. (8/6 pole, Δ/Δ , $N_s = 72$ slots)

Coil group	Slot (coil) distribution principle	Slot (coil) number
A_1	$n(t_1 = 10)$	72,1,2,3,-10,-11,-65,-66,-67,-68
B_1	$n + N_s/3$	24,25,26,27,-34,-35,-17,-18,-19,-20
C_1	$n + 2N_s/3$	48,49,50,51,-58,-59,-41,-42,-43,-44
A_2	$n'(t_2 = 10)$	21,22,23,-29,-30,-31,-32,37,38,39
B_2	$n' + N_s/3$	45,46,47,-53,-54,-55,-56,61,62,63
C_2	$n' + 2N_s/3$	69,70,71,-5,-6,-7,-8,13,14,15
a_2	$n''(t_3 = 4)$	-52,60,-9,16
b_2	$n'' + N_s/3$	-4,12,33,40
c_2	$n'' + 2N_s/3$	-28,26,-57,64

Adjusting groups of t_3 slots/phase should not produce emfs phase shifted by more than 30° with respect to groups $A_1B_1C_1$ to secure a high spread (distribution) factor.

An example of such an 8/6 pole Δ/Δ winding is shown in Table 4.1 [7]; the coil connections from table 4.1 are shown in Figure 4.25. [7]

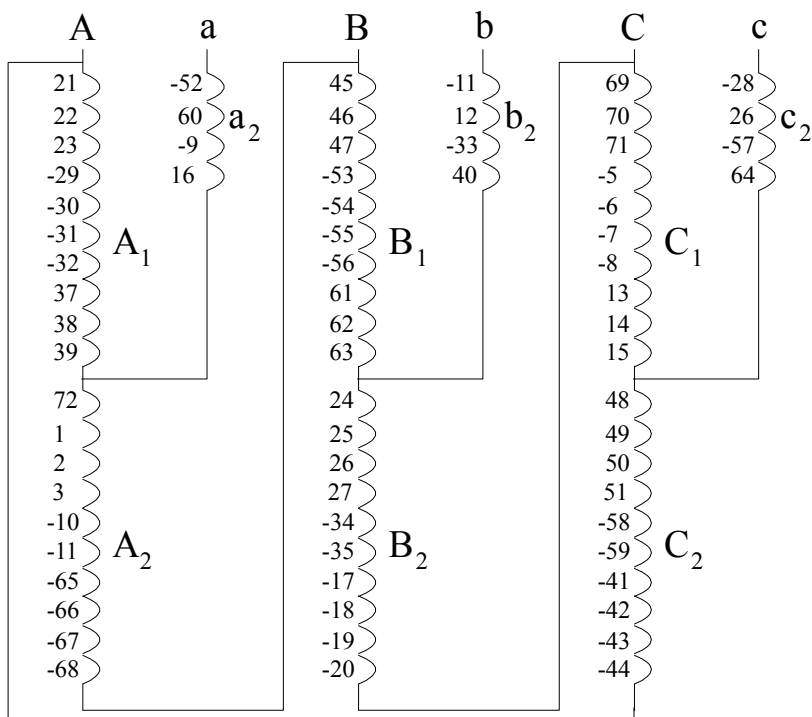


Figure 4.25 8/6 pole winding with 72 slots

Such a pole-changing winding seems suitable for fan or windmill applications as the power delivered is reduced to almost half when switching from 8 to 10 poles, that is, with a 20% reduction in speed. The procedure is similar for the $(3Y + 2Y)/3Y$ connection. [6]

This type (Figure 4.24b) for $a_1 = a_1' = b_1 = b_1' = c_1 = c_1' = 0$ and pole count ratio of say $2p_2/2p_1 = 6/2$ has long been known and recently proposed for dual winding (doubly fed) stator nest – cage rotor induction machines [16].

Once the slot/phase allocation is completed the general formula for the winding factor (4.45) or the mmf step-shape distribution with harmonics decomposition may be used to determine the mmf fundamental and harmonics content for both pole counts. These results are part of any IM design process.

➤ A pure mathematical approach to a.c. winding design

As we have seen so far, a.c. winding design is more an art than a science. Intuition and symmetry serve as tools to this aim.

Since we have a general expression for the harmonics winding factor valid for all situations when we know the number of turns per coil and what current flows through those coils in each slot, we may develop an objective function to define an optimum performance.

Maximum airgap flux energy for the fundamental mmf, or minimum of some harmonics squared winding factors summation, or the maximum fundamental winding factor, may constitute practical objective functions. Now copper weight may be considered a constraint. So is the pole count $2p_1$. Some of the above objectives may become constraints. What then would the variables be for this problem? It seems that the slot/phase allocation and ultimately, the number of turns/coil are essential variables to play with, together with the number of slots.

In a pole-changing ($2p_2/2p_1$) winding, the optimization process has to be done twice. This way the problem becomes typical: non-linear, with constraints.

All optimization methods used now in electric machine design also could apparently be applied here.

The first attempt has been made in [7] by using the “simulated annealing method”—a kind of direct (random) search method, for windings with identical coils. The process starts with fixing the phase connection: Y or Δ and with a random allocation of slots to phases A, B, C. Then we take the instant when the current in phase A is maximum i_{Max} and $i_B = i_C = -i_{\text{Max}}/2$. Allowing for a given number of turns per coil n_c and 1 or 2 layers, we may start with given (random) mmfs in each phase.

Then we calculate the objective function for this initial situation and proceed to modify the slot/phase allocation either by changing the connection (beginning or end) of the coil or its position (see Figure 4.26). Various rules of change may be adopted here.

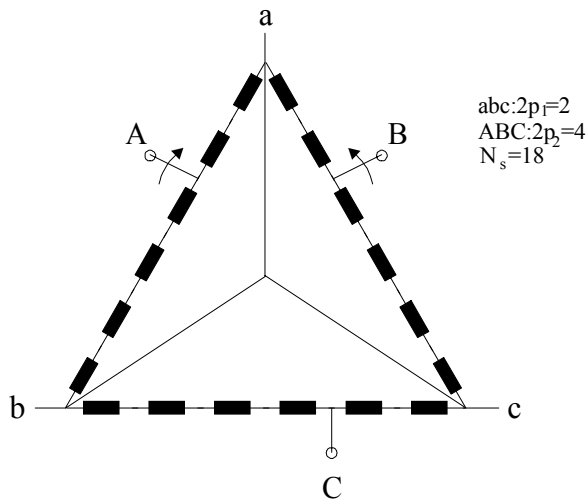


Figure 4.26 Initial (random) winding on the way to “rediscover” the Dahlander connection

The objective function is calculated again. The computation cycle is redone until sufficient convergence is obtained for a limited computation time.

In [7] by using such an approach the 4/2 pole Dahlander connection for an 18 slot IM is rediscovered after two hours CPU time on a large main frame computer. The computer time is still large, so the direct (random) search approach has to be refined or changed, but the method becomes feasible, especially for some particular applications.

4.12. THE MMF OF ROTOR WINDINGS

Induction machine rotors have either three-phase windings star-connected to three-phase slip rings or have squirrel cage type windings. The rotor three-phase windings are made of full pitch coils and one or two-layers located in semiclosed or semiopen slots. Their end connections are braced against centrifugal forces. Wound rotors are typical for induction machines in the hundreds and thousands of kW ratings and more recently in a few hundreds of MW as motor/generators in pumped storage hydro power plants.

The voltage rating of rotor windings is of the same order as that in the stator of the same machine. The mmf of such windings is similar to that of stator windings treated earlier in this chapter.

Squirrel-cage windings—single cage, double cage, deep bar, solid rotor, and nest cage types were introduced in Chapter 2. Here we deal with the mmf of a single cage winding. We consider a fully symmetrical cage (no broken bars or end rings) with straight slots (no skewing)—[Figure 4.27a](#).

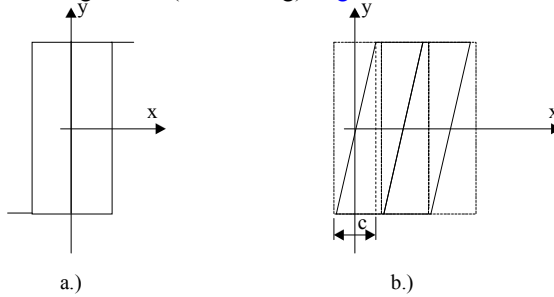


Figure 4.27 Rotor cage with; a.) straight rotor slots, b.) with skewed rotor slots

By replacing the cage with an N_r/p_1 phase winding ($m = N_r/p_1$) with one conductor per slot, $q = 1$ and full pitch coils [17], we may use the formula for the three winding (4.44) to obtain

$$F_2(\theta, t) = \frac{N_r}{\pi p_1} I_b \sqrt{2} \sum \frac{K_\mu}{\mu} \sin \frac{\nu p_1}{N_r} \cdot [K_{fv} \cos(\omega_1 t - \mu \theta) + K_{bv} \cos(\omega_1 t + \mu \theta)] \quad (4.52)$$

where I_b is the RMS bar current and

$$K_{fv, bv} = \frac{p_1}{N_s} \left[1 + \left(\frac{N_r}{p_1} - 1 \right) \cos(\mu \pm p_1) \frac{2\pi}{N_r} \right] \quad (4.53)$$

Let us remember that μ is a spatial harmonic and θ is a geometrical angle: $\theta_e = p_1\theta$. Also, $\mu = p_1$ means the fundamental of mmf.

There are forward (f) and backward (b) harmonics. Only those of the order $\mu = KN_r + p_1$, have $K_{fv} = 1$ while for $\mu = KN_r - p_1$, they have $K_{bv} = 1$. All the other harmonics have $K_{fv,bv}$ as zero. But the above harmonics are, in fact, caused by the rotor slotting. The most important, are the first ones obtained for $K = 1$: $\mu_{fmin} = N_r - p_1$ and $\mu_{bmin} = N_r + p_1$.

For many sub-MW-power induction machines, the rotor slots are skewed, in general by about one stator slot pitch or more, to destroy the first stator-slot-caused stator mmf harmonic ($v = N_s - p_1$). Again v is a spatial harmonic; that is $v_1 = p_1$ for the fundamental.

In this situation let us take only the case of the fundamental. The situation is as if the rotor position in (4.54) varies with y (Figure 4.27b).

$$F_2(\theta, t)_{v=p_1} \approx \frac{N_r}{\pi p_1} I_b \sqrt{2} \cdot \cos \left[\omega_1 t - \left(\theta_e - \frac{\pi y}{\tau} \right) - (\pi - \gamma_{1,2}) \right]; \quad \theta_e = \frac{\pi x}{\tau} \quad (4.54)$$

$$\text{for } y \in [0.0, \pm c/2]$$

Now as ω_1 , the stator frequency, was used in the rotor mmf, it means that (4.54) is written in a stator reference system,

$$\omega_1 = \omega_2 + \omega_r; \quad \omega_r = \Omega_r p_1 \quad (4.56)$$

with ω_r as the rotor speed in electrical terms and ω_2 the frequency of the currents in the rotor. The angle $\gamma_{1,2}$ is a reference angle (at zero time) between the stator $F_1(\theta, t)$ and rotor $F_2(\theta, t)$ fundamental mmfs.

4.13. THE “SKEWING” MMF CONCEPT

In an induction motor on load, there are currents both in the stator and rotor. Let us consider only the fundamentals for the sake of simplicity. With $\theta_e = \pi x/\tau$, (4.13) becomes

$$F_1(\theta_e, t) = F_{1m} \cos(\omega_1 t - \theta_e) \quad (4.57)$$

with F_{1m} from (4.14).

The resultant mmf is the sum of stator and rotor mmfs, $F_1(\theta_e, t)$, and $F_2(\theta_e, t)$. It is found by using (4.54) and (4.57).

$$F_{r1}(\theta_e, t) = F_{1m} \cos(\omega_1 t - \theta_e) + F_{2m} \cos \left(\omega_1 t - \theta_e - \frac{\pi y}{\tau} - (\pi - \gamma_{1,2}) \right) \quad (4.58)$$

$$\text{with} \quad F_{1m} = \frac{3W_1 I \sqrt{2} K_{w1}}{\pi p_1}; \quad F_{2m} = \frac{N_r}{\pi p_1} I_b \sqrt{2} \quad (4.59)$$

The two mmf fundamental amplitudes, F_{1m} and F_{2m} , are almost equal to each other at standstill. In general,

$$\frac{F_{2m}}{F_{1m}} \approx \sqrt{1 - \left(\frac{I_0}{I}\right)^2} \quad (4.60)$$

where I_0 is the no load current.

For rated current, with $I_0 = (0.7 \text{ to } 0.3)I_{\text{Rated}}$, F_{2m}/F_{1m} is about (0.7 to 0.95) at rated load. As at start (zero speed) $I_s = (4.5 \text{ to } 7)I_n$, it is now obvious that in this case,

$$\left(\frac{F_{2m}}{F_{1m}}\right)_{S=1} \approx 1.0 \quad (4.61)$$

The angle between the two mmfs $\gamma_{1,2}$ varies from a few degrees at standstill ($S = 1$) to 45° at peak (breakdown) slip S_K and then goes to zero at $S = 0$. The angle $\gamma_{1,2} > 0$ for motoring and $\gamma_{1,2} < 0$ for generating.

The resultant mmf $F_r(\theta_e, t)$, with (4.54) and (4.57), is

$$\begin{aligned} F_r(\theta_e, t) = & \left[F_{1m} \cos(\omega_1 t - \theta_e) - F_{2m} \cos(\omega_1 t - \theta_e + \gamma_{1,2}) \cos \frac{\pi y}{\tau} \right] + \\ & + F_{2m} \sin(\omega_1 t - \theta_e + \gamma_{1,2}) \cdot \sin \frac{\pi y}{\tau}; y \in (-c/2, +c/2) \end{aligned} \quad (4.62)$$

The term outside the square brackets in (4.62) is zero if there is no skewing. Consequently the term inside the square brackets is the compensated or the magnetizing mmf which tends to be small as $\cos \pi y / \tau \cong 1$ and thus the skewing $c = (0.5 \text{ to } 2.0)\tau/mq$ is small and the angle $\gamma_{1,2} \in (\pi/4, -\pi/4)$ for all slip values (motoring and generating).

Therefore, we define $F_{2\text{skew}}$ by

$$F_{2\text{skew}} = F_{2m} \sin(\omega_1 t - \theta_e + \gamma_{1,2}) \sin \frac{\pi y}{\tau}; y \in (-c/2, +c/2) \quad (4.63)$$

the uncompensated rotor-produced mmf, which is likely to cause large airgap flux densities varying along the stator stack length at standstill (and large slip) when the rotor currents (and F_{2m}) are large. Thus, heavy saturation levels are expected along main flux paths in the stator and rotor core at standstill as a form of leakage flux, which strongly influences the leakage inductances as shown later in this book.

4.14. SUMMARY

- A.C. windings design deals with assigning coils to slots and phases, establishing coil connections inside and between phases, and calculating the number of turns/coil and the wire gauge.

- The mmf of a winding means the spatial/time variation of ampere turns in slots along the stator periphery.
- A pure traveling mmf is the ideal; it may be approximately realized through intelligent placing of coils of phases in their slots.
- Traveling wave mmfs are capable producing ripple-free torque at steady state.
- The practical mmf wave has a spacial period of two pole pithces 2τ .
- There are p_1 electrical periods for one mechanical revolution.
- The speed of the traveling wave (fundamental) $n_1 = f_1/p_1$; f_1 —current frequency.
- Two phases with step-wise mmfs, phase shifted by $\pi/2p_1$ mechanical degrees or m phases (3 in particular), phase shifted by π/mp_1 geometrical degrees, may produce a practical traveling mmf wave characterized by a large fundamental and small space harmonics for integer q . Harmonics of order $v = 5, 11, 17$ are reverse in motion, while $v = 7, 13, 19$ harmonics move forward at speed $n_v = f_v/(vp_1)$.
- Three-phase windings are built in one or two-layers in slots; the total number of coils equals half the number of stator slots N_s for single-layer configurations and is equal to N_s for two-layer windings.
- Full pitch and chorded coils are used; full pitch means π/p_1 mechanical radians and chorded coils means less than that; sometimes elongated coils are used; single-layer windings are built with full pitch coils.
- Windings for induction machines are built with integer and, rarely, fractional number of slots/pole/phase, q .
- The windings are characterized by their mmf fundamental amplitude (the higher the better) and the space harmonic contents, (the lower, the better); the winding factor K_{wv} characterizes their performance.
- The star of slot emf phasors refers to the emf phasors in every slot conductor drawn with a common origin and based on the fact that the airgap field produced by the mmf fundamental is also a traveling wave; so, the emfs are sinusoidal in time.
- The star phasors are allocated to the three-phases to produce three resultant phasors 120° (electrical) apart; in partly symmetric windings, this angle is only close to 120° .
- Pole-changing windings may be built by changing the direction of connections in half of each phase or by dividing each phase into a few sections (multiples of 3, in general) and switching them from one phase to another.
- Pole-changing is used to modify the speed as $n_1 = f_1/p_1$.
- New pole-changing windings need only two single throw switches while standard ones need more costly switches.
- Single phase supplies require two-phase windings—at least to start; a capacitor (or a resistor at very low power) in the auxiliary phase provides a traveling mmf for a certain slip (speed).

- Reducing harmonics content in mmfs may be achieved by varying the number of turns per coil in various slots and phases; this is standard in two-phase (single-phase supply) motors and rather new in three-phase motors, to reduce the noise level.
- Two phase (for single phase supply) pole-changing windings may be obtained from three-phase such windings by special connections.
- Rotor three-phase windings use full pitch coils in general.
- Rotor cage mmfs have a fundamental ($\mu = p_1$) and harmonics (in mechanical angles) $\mu = kn_r \pm p_1$, N_r – number of rotor slots.
- The rotor cage may be replaced by an N_r/p_1 phase winding with one conductor per slot.
- The rotor mmf fundamental amplitude F_{2m} varies with slip (speed) up to F_{1m} (of the stator) and so does the phase shift angle between them $\gamma_{1,2}$. The angle $\gamma_{1,2}$ varies in the interval $\gamma_{1,2} \in (0, \pm \pi/4)$.
- For skewed rotor slots a part of rotor mmf, variable along stator stack length (shaft direction), remains uncompensated; this mmf is called here the “skewing” mmf which may produce heavy saturation levels at low speed (and high rotor currents).
- The sum of the stator and rotor mmfs, which solely exist in absence of skewing constitutes the so called “magnetisation” mmf and produces the main (useful) flux in the machine.
- Even this magnetising mmf varies with slip (speed) – for constant voltage and frequency; however it decreases with slip (reduction in speed); this knowledge is to be used in later Chapters (5, 6, 8).

4.15. REFERENCES

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